A Literature Survey on Different Algorithms of LMS-MVDR, LMS-MUSIC Estimation

Nida khan, Prof. Kamal Niwaria2

M-TechScholar1, Assistant Professor2

Email – kh.nidakhan@gmail.com, kamalniwaria@gmail.com 2

Department of Electronics & Communication Engineering

RKDF INSTITUTE OF SCIENCE &TECHNOLOGY , SRK UNIVERSITY.BHOPAL (M.P.)

Abstract - The **Smart antenna** systems plays an important role in wireless communications systems. This paper begins with a brief introduction about a smart antenna and majorly in this paper different type of DOA algorithms. Direction-of-Arrival (DOA) estimation plays a significant role in several applications. Beamforming is that the most prominent technique to estimate DOA. During this survey, a study of varied beamforming techniques and algorithms to estimate the direction of arrival of a signal is formed. An assessment on the background sturdy algorithms using Nyquist rate and its Compressive sensing different is completed. It’s famed that Bearing estimation algorithms acquire only a small number of direction of arrivals (DOAs) among the entire angle domain, once the sources are spatially distributed. Hence, it should be ended that, the ways those specifically exploits this spatial sparsely property is advantageous. These ways use a very small variety of measurements within the form of random projections of the sensing element knowledge beside one full wave form recording at one amongst the sensors.

Keywords-Smart Antenna, DOA, MUSIC and MVDR.

1. Introduction

In recent years the high demand on the usage of the wireless communication system has put more emphasize on the requirement of higher system capacities. The system capacity can be improved by either enlarging its frequency bandwidth or adding new range of frequency spectrum to wireless services. But because of obvious reasons, since the electromagnetic spectrum is a limited resource, it is not easy to get new spectrum allocation without the international coordination on the global level. So, one alternative approach is to use existing spectrum more efficiently. The drawback of estimating the frequency or angle of arrival of a plane wave is referred to as direction finding or DOA estimation problem. it's an oversized application in microwave radar, sonar, seismic systems, surveillance, diagnosing and treatment, radio pseudoscience and different areas. Because of its widespread applications and issue of getting an optimum calculator, the subject has a received a big quantity of attention over the last many decades. many ways exist to address the matter of estimating the direction-of-arrivals (DOAs) of multiple sources mistreatment the signals received at the sensors the applying of the array process needs either the data of a reference signal or the direction of the required signal source to attain its desired objectives. Antenna arrays are wide accustomed solve direction finding.

Beamforming is employed along with an array of antennas/sensors to transmit/receive signals to/from a specified spatial direction within the presence of interference and noise. Therefore, it acts as a spatial filter [1]. it's a classic yet continuously developing field that has practical sensible applications. Within the last decade, there has been revived interest in beamforming driven by applications in wireless communications, wherever multi-antenna techniques have emerged in concert of the key technologies to accommodate the explosive growth of the number of users and quickly increasing demands for brand new high data-rate services.

The techniques for estimating the directions of arrival of signals victimization an antenna array is booming in recent years. Many ways in which exist and are classified in step with the technique used, the info they have (external or not) and eventually the criterion used (conventional ways in which, projection on the noise or offer topological space, most likelihood technique, etc. A receive beamformer is typically accustomed estimate the signal returning from a particular direction among the presence of noise and meddling signals. throughout a receive beamformer, the output of the array of sensors are linearly combined pattern abstraction filter coefficients (weight vector) so as that the signals arriving from a desired direction square measure passed to the beamformer output ingenuous, whereas signals from totally different directions square measure attenuated.

With a central focus on bearing estimation, the prime objective here is to seek out the supply of transmitted communication/ measuring system signal. Through this paper, associate comprehensive literature survey is made on the various bearing estimation techniques and algorithms till date. These many estimation algorithms additionally square measure available among the literature have utterly totally different capabilities and limitations [2]-[3]. The DOA estimation disadvantage in some cases, is initial solved by estimation ways in which of the quantity of the reformer [4]-[5]-[6] so applying a high-resolution technique to estimate the relation of these sources. These high-resolution ways in which area unit known to be stronger than embrace techniques. the foremost general beam-forming techniques embody normal any as adjective beamformers. For the normal non-adaptive beamformers, the load vector for a specific direction of arrival (DOA) depends on the array response alone and may well be pre-calculated, freelance of the received data. Thus, they are data freelance beamformers which they gift a relentless response for all signal/interference things. The adjuster beamformers square measure information-dependent since the load vectors are calculated as perform of the incoming knowledge to optimize the performance subject to various constraints [2]. They have higher resolution and much higher interference rejection capability than the data-independent beamformers.

1. **Antenna Array**

Smart antennas are based on using multiple antenna elements configured in an array. The configuration of the array has a direct effect on the performance of the system. There is a variety of antenna array configurations that have been used in smart antenna systems over the past decades including linear, rectangular, and circular. The choice of antenna array configuration depends on the desired specifications of the system which include cost, number of users, accuracy, range, steering, and noise cancellation.

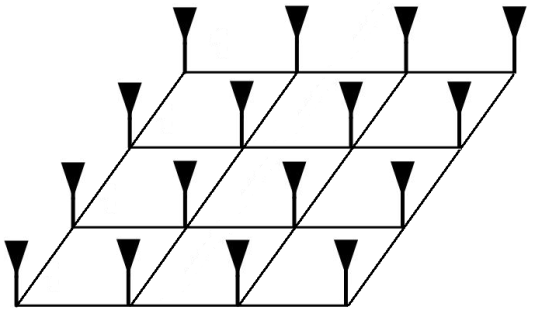


Figure 1 16-element uniform rectangular array.

# **BEAMFORMING**

A conventional beamformer features a structure shown in Figure one, wherever the weights are pre-calculated and are freelance of the incoming information. The weights are essentially the delay encountered in every detector attributable to path distinction, in order that the outputs of spatially distributed sensors area unit coherently summed to boost signal reception within the presence of noise. contemplate a linear array of M sensors, with a regular put down detector spacing (d). The detectors spatially sample the signal field at the sensor locations.

1. **Adaptive Beamformer**

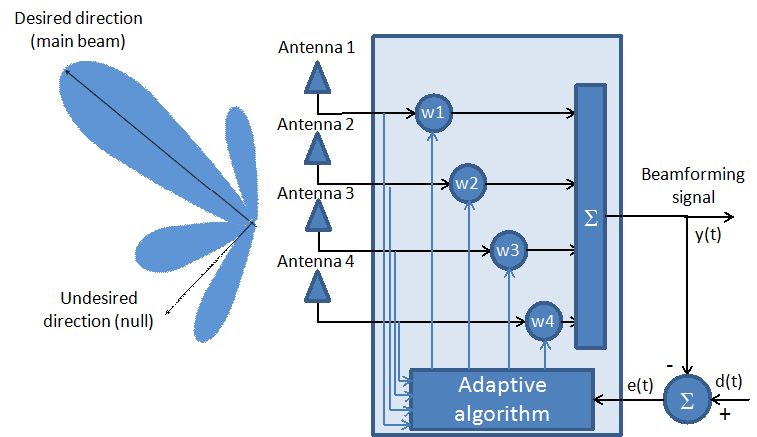
Adaptive beamforming is employed for enhancing a desired signal whereas suppressing noise and interference at the output of an array of sensors. The aim of the adaptation beamforming is to optimize a group of weight vectors to find a directional supply. There are a unit completely different ways in inbound at this improvement downside. Figure a pair of shows the structure of AN adaptive beamformer. In applications wherever, signal strength is unknown and is often gift, application of linear constraints to the load vector permits in depth management of the adaptive behavior of the beamformer.

Figure Adaptive Beam Former

1. Literature Survey

As A survey of beamformers together with adaptational beamformers is given in [1]. The conditions below that the adaptational beamformer performance degrades are seen in several papers. There exist variety of techniques to estimate the DOA of signals of interest. Here, a survey on ordinarily used techniques moreover as algorithms is created. In general, the DOA estimation adaptational beamforming algorithms could also be classified into Beam scan Algorithms, and Beam space algorithms [2].

**Diagonal Loading -** Among the various sturdy adaptational beamformers projected within the literature, diagonal loading emerges because the most generally used methodology owing to its simplicity and its effectiveness in handling a good sort of errors, together with steering vector and finite-sample errors [4]. it's sturdy against finite sample errors [5]. However, a significant downside of the diagonal loading technique is that there's no reliable way to choose the diagonal loading issue, that directly affects its performance.

**Eigen space Based Technique** - Another fashionable sturdy adaptational beamforming technique is that the eigenspace-based beamformer [6]. The key plan of this method is to project the signal steering vector onto the calculable signal-plus-interference topological space obtained via the Manfred Eigen decomposition of the sample variance matrix. If the rank of signal-plus-interference topological space is low and if the amount of interference directions, L area unit precisely renowned, the eigenspace-based beamformer is understood to supply glorious lustiness against discretional steering vector errors. sadly, this approach might degrade severely if the low-rank interference-plus-signal assumption is desecrated or if the topological space dimension L is unsure or renowned inexactly. as an example, within the presence of incoherently scattered (spatially dispersed) officious sources, interferers with at random unsteady wave fronts, and moving interferers, the low rank interference assumption might become violate to might move aloof from the sharp notches of the custom-made pattern, and this could cause a powerful degradation of the output Signal-to-Interference and Noise ratio (SINR). An economical remedy for adaptation array performance in such things relies upon artificial broadening of the null breadth toward the directions of officious sources victimization spinoff constraints [9], [10].

A robust beamformer for the foremost general case of associate discretional dimension of the required signal topological space is developed in [11], and is applicable to each the rank-one and better rank desired signal models. The projected sturdy adaptational beamformers area unit supported express modeling of uncertainties within the desired signal array response and information variance matrix moreover as worst-case performance improvement. Closed type solutions and computationally economical on-line implementations of the sturdy rule are developed in [11].

Capon Beamforming - In [12], the sturdy Capon beamformer is projected, wherever the variance fitting formulation of the quality capon beamformer, is in addition to the constraint that the beamformer response be higher than some level for all the steering vectors that lie associate ellipsoid (sphere) centered on the nominal or plausible steering vector of interest. In [13], a further norm constraint is additionally wont to get the doubly unnatural sturdy Capon Beam former. A computationally economical sturdy adaptation beamforming theme is developed in [14], to account for the signal array response twin and tiny coaching sample size. It includes a quadratic difference constraint and is enforced with gradient descent methodology. All the sturdy adaptation algorithms, surveyed until currently, was for narrowband signals. However, in several applications, the signals are band and thence sturdy band adaptation algorithms are essential. The foremost fashionable approach within the style of band adaptation beamformers is to decompose the received broadband signals into narrowband parts and so to use separate narrowband beamformers to every sub band [15].

Tapped electrical circuit Beamformer - associate alternate approach within the style of wide band beamformers is to use broached delay-lines (TDLs) [16], which may type a frequency dependent response for every of the received broadband detector signals to compensate the section distinction for various frequency parts. a sturdy rule for broadband arrays was projected in [17] victimization worst case improvement, wherever a gaggle of constraints are obligatory on sampled frequency points over of constraints are imposed on sampled frequency points over the frequency range of interest to prevent the mismatched desired signal.

Chhaya Singh, B G Hogade “Implementation of an Adaptive Beam Forming Antenna for Radio Technology”- In this analysis article authors says that Beamforming antennas for fastened and mobile wireless communications have received monumental interest worldwide in recent decades, and a large type of approaches for good antenna style and application. good antenna techniques at the bottom station will dramatically improve the performance of the mobile radio system by using abstraction filtering. The band good antennas area unit wide used antennas. A band beamforming algorithmic program that springs from abstraction signal process technique is considered during this technique. we'll be exploitation circular array pure mathematics for the band good antenna. a well-known LMS algorithmic program are going to be applied to the circular array pure mathematics. The DOA/validation element uses a MATLAB script to implement the MUSIC algorithmic program to estimate the DOA for each incoming source. during this paper directional beam pattern for the given style parameters are going to be displayed.

Bindu Sharma, Indranil Sarkar “Performance Analysis of Smart Antenna Beam forming Techniques “The wireless cellular base station antenna system employs switched beam technology that suffers from its unskillfulness to trace the user and restricted capability. The good antenna tracks the mobile user a lot of expeditiously by guiding the most beam towards the user and forming nulls within the directions of the meddling signal. Good antennas embrace the look of antenna array and adjusting the incoming signal by dynamical the weights of the amplitude and section exploitation economical DSP algorithms [1]. This paper chiefly focuses on the adaptive beam forming algorithms like LMS, SMI, RLS, CGA, CMA and LSCMA applied for uniform linear array antenna. The higher than adaptive algorithms are simulated exploitation MATLAB.

# Do a Estimation Algorithms

The algorithms supported DOA are classified as non-subspace or quadratic sort and mathematical space sort [6]. The Bartlett and Capon (Minimum Variance Distortion Less Response) [6] are quadratic sort algorithms. each the ways area unit extremely addicted to physical size of array aperture, which ends up in poor resolution and accuracy, [5], [7], [9], [11], [12], [13]. Mathematical space primarily based DOA estimation technique relies on the chemist decomposition [8]. The mathematical space primarily based DOA estimation formula MUSIC provides high resolution, and is additional correct and not restricted to physical size of array aperture [2] [7]. The assorted DOA formula performance is analyzed supported variety of snapshots, variety of users, user house distribution, variety of array components, SNR and MSE.

Algorithms for estimating DOA may be classified into beams will algorithms and mathematical space algorithms [2]. The beams will algorithms type a standard beam, scans it over the suitable region and plots the magnitude square of output. This estimator is noted because the Bartlett beamformer [20-21]. MVDR, Root MVDR are the samples of this.

Subspace algorithms, are a group of algorithms, whereby the orthogonality between the signal and noise subspaces is exploited [2]. These are noted as high resolution mathematical space primarily based algorithms. the fashionable high-resolution ways supported the construct of mathematical space, like MUSIC, Root-MUSIC related to. Their advantage is that subspaces solely rely upon the pure mathematics of the network and therefore the position of sources.

MVDR Algorithms - In Minimum Variance Distortion Less Response beamformer the linear filter weights employed in the beamformer are adaptively calculated reckoning on the atmosphere therefore on suppress the interferences to the utmost, exploit the signal of interest in generous [22]. Here the computation of the inverse matrix and its multiplication with steering vector area unit the foremost necessary components within the method of best weight computation. The array matrix (R) could be a live of the special correlation of the signal and noise inbound at the array. accommodative beam forming techniques measures the array matrix rather than forward that the noise is white and Gaussian. This array matrix mensuration is then accustomed confirm the special filter coefficients (weights). MVDR shows degraded performance compared to standard beamformers, once there's position errors in sensors.

Root MVDR - Performs moderately well higher than threshold, however threshold is beyond most probability algorithms closely spaced signals. However, the threshold of MVDR formula is beyond root MVDR [2]. it's used as a preliminary processor to point the quantity of plane waves natural event on the array, their approximate location, and approximate signal power. but it suffers a demerit that, just in case of two closely spaced plane waves, algorithms can suppose they're single plane waves and underestimate the quantity of signals.

The performance of the MVDR beamformer is severely tormented by the correlation between the look-direction signal and therefore the interferences. special smoothing could be a technique accustomed alleviate the issues owing to correlation wherever the array is split into smaller sub arrays, and therefore the average of the all the sub array variance matrices are accustomed type a smoothened R matrix. In [23],

In the recent past, some strong algorithms with clear theoretical background are planned that build express use of associate uncertainty set of the array steering vector. In [25], spherical uncertainty sets are used and in [26] spheroidal (including flat ellipsoidal) uncertainty sets are used. Here, the beamformer is intended to reduce the output power subject to the constraint that the beamformer response is higher than some level for all the steering vectors that exist associate ellipsoid (sphere) targeted on the nominal or likely steering vector of interest. This guarantees that the signal of interest, whose steering vector is anticipated to exist the ellipsoid (sphere), won't be eliminated, and hence, hardiness is improved. once the ellipsoid could be a sphere, then the answer to the preceding drawback is of the diagonal loading sort, wherever the loading level is obtained victimization the variance matrix and therefore the radius of the sphere. within the case wherever the ellipsoid isn't a sphere or is flat, the strong beamformer takes the shape of a general (i.e., not essentially diagonal) loading of the variance matrix.

MUSIC Algorithms - Multiple Signal Classification formula [27-28] uses the eigenvectors decomposition and eigenvalues of the variance matrix of the antenna array for estimating directions-of-arrival of sources supported the properties of the signal and noise subspaces. Many variants of MUSIC like Spectral, Unitary, and Root MUSIC ways are planned to cut back complexness, increase performance and backbone power. The advantage of Root Music is that the direct calculation of the DOA by the hunt for zeros of a polynomial, that replaces the hunt for maxima [29], necessary within the case of MUSIC. This technique is restricted to linear antennas uniformly spaced out. However, it permits a discount in computing time then a rise within the angular resolution by exploiting sure properties of the received signals. The principle of the Root-

Compression of Different Algorithm

|  |  |  |
| --- | --- | --- |
| Eigen-Space Based | Excellent robustness against arbitrary steering vector errors. | Degrade severely if the low rank interference-plus-signal assumption is violated or if the sub space dimension*l*ess uncertain or knownimprecisely |
| LCMV | Improved robust ness | Strong degradation of the output SINR |
| MVDR-  Minimum Variance Distortion Less Performance | Gives distortion less performance in the Direction of Interest | Unable to distinguish between wo closely space planewaves |
| Root MVDR | Better Performance | Lesser threshold compared to MVDR |
| MUSIC-  Multiple Signal Classification Algorithm | High-level of orthogonality between signals  Higher resolution&accuracy | Gives the Pseudo spectrum only. |
| Root MUSIC | Less  computational time, higher resolution | Limited to linear antennas, spaced. |
| Unitary MUSC | Less Computational complexity | No much performance improvement from Root MUSIC. |
| ESPIRIT-  Estimation of signal parameters via rotational in variance technique | No need of searching the maxima in Pseudospectrum.  Less sensitive to noise | More prone to errors. |

# Conclusion And Future Work

This paper shows the analysis of direction of arrival (DOA) estimation using MUSIC and MVDR algorithms. Estimating the direction-of arrival (DOA)of propagating plane waves is a problem of broad interest in a variety of fields including wireless communications, radar and sonar systems, acoustic signal processing, medical imaging and seismology. Through this review, a detailed survey on various DOA estimation beam forming algorithm existing, were made. Based on the literature survey made, it maybe I concluded that beam forming based on Compressive Sensing for DOA estimation Is more advantageous. As the signal of interest, here is a sparse signal, it is advisable to switch onto Compressive sensing-based beam forming for DOA estimation which requires only a fewer number of samples, rather than the Nyquist sampling. Compressive sensing-based beam forming allied Compressive Beam forming for DOA estimation was found to be more beneficial over Nyquist sampling.

.

# References

[1] Barry D. Van Veen and Kevin M. Buckley, “Beamforming: A Versatile Approach to Spatial Filtering”, IEEE ASSP magazine April 1988.

[2] H. Van Trees, Optimum Array Processing. New York: Wiley, 2002, sec. Part IV of “Detection, Estimation and Modulation Theory”.

[3] Mestre and Lagunas’: “Finite Sample Size Effect on Minimum Variance Beamformers: Optimum Diagonal Loading Factor for Large Arrays” IEEE Trans. Signal Processing, Vol. 54, No. 1, Jan 2006.

[4] H. Cox, R. M. Zeskind, and M. H. Owen, “Robust adaptive beamforming,” IEEE Trans. Acoust., Speech, Signal Processing, vol. 35, Oct. 1987.

[5] B. D. Carlson, “Covariance matrix estimation errors and diagonal loading in adaptive arrays,” IEEE Trans. Aerosp. Electron. Syst., vol. 24, July 1988.

[6] L. Chang and C. C. Yeh, “Performance of DMI and eigenspace-based beamformers,” IEEE Trans. Antennas Propagation, vol. 40, Nov. 1992.

[7] M.H. Er and A. Cantoni, “Derivative constraints for broadband element space antenna array processors,” IEEE Transactions on Antennas and Propagation, vol. ASSP-31, no. 6, December 1983.

[8] O. L. Frost III, “An algorithm for linearly constrained adaptive processing,” Proc. IEEE, vol. 60, Aug. 1972.

[9] A.B. Gershman, G.V. Serebryakov, and J. F.Bohme, “Constrained Hung-Turner adaptive beamforming algorithm with additional robustness to wideband and moving jammers,”IEEE Transactions on Antennas and Propagation, vol. AP-44,March 1996.

[10] R. Li, X. Zhao, and X. W. Shi “Derivative Constrained Robust LCMV Beamforming Algorithm” , Progress In Electromagnetics Research C, Vol. 4, 2008.

[11] S. Shahbazpanahi, A. B. Gershman, Z.-Q. Luo, and K. M. Wong, “Robust adaptive beamforming for general-rank signal models,” IEEE Trans. Signal Processing,Vol. 51, Sept. 2003.

[12] R. G. Lorenz and S. P. Boyd, “Robust minimum variance beamforming,” IEEE Trans. Signal Processing, vol. 53, May 2005.

[13] J. Li, P. Stoica, and Z. Wang. On robust capon beamforming and diagonal loading. IEEE Trans. SignalProc., 51, July 2003.

[14] J. Li, P. Stoica, and Z. Wang. Doubly constrained robust capon beamformer. IEEE Trans. Signal Processing, 52(9): Sep. 2004.

[15] Xin Song, Jinkuan Wang, Bin Wang, Yinghua Han, “Robust Adaptive Beamforming under in the Presence of Mismatches”, Proceedings of the IEEE International Conference on Automation and Logistics, August 2009.

[16] E. W. Vook and R. T. Compton, Jr., “Bandwidth performance of linear adaptive arrays with tapped delay-line processing,” IEEE Transactions on Aerospace and Electronic Systems, vol. 28, no. 3, July 1992.

[17] M. Rubsamen and A.B. Gershman, “Robust presteered broadband beamforming based on worst-case performance optimization,” in Proc. IEEE Workshop on Sensor Array and Multichannel Signal Processing, July 2008.

[18] Yong Zhao, Wei Liu; Langley, R.J, “Robust Broadband Beamforming Based on Frequency Invariance Constraints and worst-case Performance Optimization”, Proceedings of the 4th International Symposium on Communications, Control and Signal Processing, March 2010.

[19] Yong Zhao, Wei Liu; Langley, R.J, “Adaptive Wideband Beamforming With Frequency Invariance Constraints”, IEEE Transactions on Antennas and Propagation, Volume:59, Issue 4,April 2011

[20] J. Capon, “.High resolution n frequency wavenumber spectrum nalysis” Proc. IEEE, August 1969.

[21] R. T. Lacoss. Data adaptive spectral analysis methods. Geophysics, ~01.36, p. 661, 1971.

[22] Vaidhyanathan & Buckley, “ Performance analysis of MVDR spatial spectrum estimator”, IEEE transactions on Signal processing, June 1995.

[23] V U. Reddy, A. Paulraj, and T. Kailath, “Performance analysis of the optimum beamformer in the presence of correlated sources and its behavior under spatial smoothing,” IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-35, July 1987.

[24] K J. Raghunath, and V. U Reddy, “Finite Data Performance Analysis of MVDR Beamformer with and without Spatial Smoothing” IEEE Transactions On Signal Processing, Vol. 40, NO. 2, Nov 1992.

[25] Vorobyov, S., Gershman, A., and Luo, Z.: ‘Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem’, IEEE Trans. Signal Process., vol 51,No.2, , Feb 2003.

[26] S. A. Vorobyov, A. B. Gershman, Z-Q. Luo, and N. Ma, “Adaptive beamforming with joint robustness against mismatched signal steering vector and interference nonstationary”IEEE Signal Processing Lett., vol. 11, Feb. 2004.

[27] R. 0. Schmidt, “Multiple emitter location and signal parameter estimation” IEEE Trans. Antennas Propag., vol.AP-34, pp. 276-280, March 1986.

[28] G. Bienvenu and L. Kopp., “Optimality of high resolution array processing using the eigensystem approach”, IEEE Trans. Acoust., Speech, Signal rocess, vol.ASSP-31, pp. 1234-1248, October 1983.

[29] B. D. Rao and K. V. S. Hari, “ Performance analysis of root-MUSIC” IEEE Trans. Acous t., Speech, Signal Process., vol.ASSP-37, pp. 1939-1949, December 1989.

[30] M. Pesavento, A. B. Gershman, and M. Haardt, “.Unitary MUSIC with real valued eigen decomposition: A theoretical and experimental study”, IEEE Trans, signal process, May 2000

[31] R. Roy and T. Kailath, “ ESPRIT: Estimation of signal parameters via rotational invariance techniques” , IEEE Trans. Acoust., Speech, Signal Process, vol.ASSP-37, pp. 984-995, July 1989.

[32] R. Roy, “ ESPRIT: Estimation of Signal Parameters via Rotational Invariance Techniques”, Ph.D. Dissertation, Stanford University, Stanford, California, 1987.

[33] X. L. Xu and K. M. Buckley, “ Bias analysis of the MUSIC location estimator”, IEEE Trans. Acoust., Speech, Signal Process., vol.ASSP-40, pp. 2559-2569, October 1992.

[34] Lotfi Osman, “ Comparative Study of High-Resolution Direction-of-Arrival Estimation Algorithms for Array Antenna System”, International Journal of Research and Reviews in Wireless Communications (IJRRWC) Vol. 2, No. 1, March 2012.

[35] R. Roy, A. Paulraj, and T. Kailath,. “ ESPRIT-A subspace rotation approach to estimation of parameters of cisoids in noise”, *IEEE Trans. Acoust., Speech, Signal Process., vol.ASSP-34, pp. 1340-1342*, October 1986.

[36] R. Roy and T. Kailath, “ESPRIT-Estimation of signal parameters via rotational invariance techniques”, In E. F. Deprettere, editor, Singular Value Decomposition and Signal Processing North-Holland, Amsterdam, The Netherlands, September 1987.

[37] Jihao Yin and Tianqi Chen, “Direction of Arrival Estimation Using a Sparse Representation of Array Covariance Vectors”, IEEE Transactions on Signal processing, Vol. 59, No. 9, September 2011.

[38] D Malioutov, M Cetin and AS Willsky, “ A Sparse signal reconstruction perspective for source localization with sensor arrays”, *IEEE Transactions on Signal processing, vol. 53,no 8*, Aug 2005.

[39] D. Donoho, “Compressed sensing”, IEEE Trans. on Information Theory, pp. 1289-1306, Apr. 2006.