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**APPLICATION OF QUEUING THEORY TO PASSENGERS’ DEPARTURE**

**ABSRACT**: Waiting in line is a very volatile situation in life which causes unnecessary delays and reduces the service effectiveness of an establishment. Queuing is a common phenomenon which occurs whenever the current demand of a service exceed the current capacity to provide for the service. This imbalance may only be temporary but during that temporary imbalanced queuing is formed. this study focus on determine the randomness of the arrival and departure of the departing passengers (passenger traveling out) at Murtala Muhammed International Airport (MMIA), Lagos Nigeria and a case study of Qatar Airways was used and the study also focus on mean arrival rate, average time a passenger spends in the system, number of passenger in the system and on the queue, Idle time of the airport attendant and traffic intensity, and study use observational method in collection of data recorded between the hours of 08:00 am and 12:00 pm at the Muritala Muhammed Airport, Lagos and it was concluded that that there exist no or less queue at the arrival counter on Qatar airway in Muritala Muhammed Airport, Lagos**.** Passengers are attended to almost immediately on their arrival. The maximum time a passenger spends on the system is few. The servers are efficient enough. For a more efficient service level, the number of servers could be increased.

KEYWORDS: Queuing theory, waiting time, Poisson arrival, exponential service times

**INTRODUCTION**

Queuing theory is the mathematical theory of the formation and behavior of queues or waiting line being on a queue can sometimes be very frustrating and over the years queuing has been a great problem to the services rendering departments. Not only is it a problem to the institutions rendering these services, it is also a problem to the customers (passengers) to whom the service is rendered (since an average service seeker would not like the ideal of waiting on an endless queue before he/she is rendered the service he seeks). The airport being one of the services rendering out, it is not left out of this challenge. Some research as work on queue in hospital, bank, market and industries. Adeleke et al.(2009) worked on the application of queuing theory to waiting time of out-patients in hospitals The average number of patients, the average time spent by each patient as well as the probability of arrival of patients into the queuing system was obtained. Ogunwale and Olubiyi (2012) studied the comparative Analysis of Waiting Time of Customers in banks, Tsarouhas (2011) presented queuing theory to calculate the total processing time per pizza line at work station in food production line, Kumar and Jain (2013) studied threshold F- policy and N- policy for controlling the arrivals and service in the queuing Bakari et al. in (2014). Used supermarket, queuing process and its application to customer service delivery was proposed Azmat (2007) revealed the use of queuing theory in the analysis of the sales check out operation in a supermarket, Houda and Taoufik (2011) in there study used simulation of queuing theory in the toll motorway, Animatu et al. (2018) studied how the use of queuing theory in vehicular traffic flow can help in minimizing the delay on roads in the Kumasi, In 2014, Prasanta Kumar Brahma worked on using queuing theory and simulation model to optimize hospital central laboratory sample collection room. Afrane and Appah (2014) worked on queuing theory and the management of waiting time in hospitals Anglo Gold Asha.nti Hospital in Ghana. The study investigated the application of queuing theory and modelling to the queuing problems at the out- patient department of the hospital., Ademoh and Nneka (2014). worked on BDR modeling of passenger queues at Nnamdi Azikiwe International Airport, Abuja, Nigeria. The study developed a queuing model using the birth and death rate approach to simulate model for validation. Balakrishnan and Simaiavis (2015) worked on a queuing model of the airport departure process. This will focused on determine the randomness of the arrival and departure of the departing passengers.

 Methods

**QUEUING EQUATION FOR A SINGLE CHANNEL, POISSON ARRIVAL AND EXPONENTIAL SERVICE TIMES (M/M/1)**

**FOR EVENT A**

|  |  |  |
| --- | --- | --- |
| A | Events | Probability |
| 1 | At time t, if (n-1) units are in the queue | $$p\_{n-1}(t)$$ |
| 2 | During the time ∆t, only one unit arrives | $$λ(∆t)$$ |
| 3 | During the time ∆t, if no units is serviced | $$1-µ(∆t)$$ |

Multiplying the three probabilities,

$$p\_{n-1}(t)×λ(∆t)×[1-µ(∆t)]$$

$$p\_{n}\left(t+∆t\right)=p\_{n-1}(t)[λ\left(∆t\right)-λµ(∆t)^{2}]$$

Since $(∆t)^{2}$ → 0 as t →∞ we have;

$$p\_{n}\left(t+∆t\right)=p\_{n-1}(t)[λ\left(∆t\right)-λµ\left(0\right)=p\_{n-1}\left(t\right)λ\left(∆t\right) (1)$$

 **FOR EVENT B**

|  |  |  |
| --- | --- | --- |
| B | Events | Probability |
| 1 | At time t, if (n+1) units are in the queue | $$p\_{n+1}(t)$$ |
| 2 | During the time ∆t, if one is serviced | µ(∆t) |
| 3 | During the time ∆t, if no units arrives | $$1-µ(∆t)$$ |

Multiplying the three probabilities,

$$p\_{n+1}(t)×µ(∆t)×[1-µ(∆t)]$$

$$p\_{n}\left(t+∆t\right)=p\_{n+1}(t)[µ\left(∆t\right)-λµ(∆t)^{2}]$$

Since $(∆t)^{2}$ → 0 as t →∞ we have;

$$p\_{n}\left(t+∆t\right)=p\_{n+1}\left(t\right)µ\left(∆t\right) (2)$$

**FOR EVENT C**

|  |  |  |
| --- | --- | --- |
| C | Events | Probability |
| 1 | At the t, if there are n units in the queue | $$p\_{n}(t)$$ |
| 2 | During the time $∆t$, one unit arrives | $$λ(∆t)$$ |
| 3 | During the time $∆t$, one unit is serviced | $$µ(∆t)$$ |

Multiplying the three probabilities,

$$p\_{n}(t)×λ(∆t)×µ(∆t)$$

Since $(∆t)^{2}$ → 0 as t →∞ we have;

$$p\_{n}\left(t+∆t\right)=p\_{n}\left(t\right)λµ\left(∆t\right)^{2}=0 (3)$$

**FOR EVENT D**

|  |  |  |
| --- | --- | --- |
| D | Events | Probability |
| 1 | T time t, if there are n units in the queue | $$p\_{n}(t)$$ |
| 2 | During the time $∆t$, | $$1-λ(∆t)$$ |
| 3 | During the time $∆t$, | $$1-µ(∆t)$$ |

Multiplying the three probabilities,

$$p\_{n}(t)×[1-λ(∆t)]×[1-µ(∆t)]$$

$$p\_{n}(t)[1-µ\left(∆t\right)-λ\left(∆t\right)+µλ(∆t)^{2}]$$

Since $(∆t)^{2}$ → 0 as t →∞ we have;

$$p\_{n}\left(t+∆t\right)=p\_{n}(t)[1-(µ+λ)(∆t)]$$

$$p\_{n}\left(t+∆t\right)=p\_{n}\left(t\right)-p\_{n}\left(t\right)\left(µ+λ\right)∆t (4)$$

Adding the total probabilities for A, B, C, D.

$$p\_{n}\left(t+∆t\right)=p\_{n-1}\left(t\right)λ\left(∆t\right)+p\_{n+1}\left(t\right)µ\left(∆t\right)+0+p\_{n}\left(t\right)-p\_{n}\left(t\right)\left(µ+λ\right)∆t$$

$$p\_{n}\left(t+∆t\right)+p\_{n}\left(t\right)=p\_{n-1}\left(t\right)λ\left(∆t\right)+p\_{n+1}\left(t\right)µ\left(∆t\right)+p\_{n}\left(t\right)-p\_{n}\left(t\right)\left(µ+λ\right)∆t$$

Dividing by$(∆t)$,

$$\frac{p\_{n}\left(t+∆t\right)+p\_{n}\left(t\right)}{∆t}=λp\_{n-1}\left(t\right)+µp\_{n+1}\left(t\right)-p\_{n}\left(t\right)\left(µ+λ\right)$$

Taking the limit as $(∆t)\rightarrow 0$

 $δ\frac{p\_{n}(t)}{δt}=λp\_{n-1}\left(t\right)+µp\_{n+1}\left(t\right)-p\_{n}\left(t\right)\left(µ+λ\right) $

At n=0

$$δ\frac{p\_{o}\left(t\right)}{δt}=µp\_{1}\left(t\right)-p\_{o}\left(t\right)-p\_{0}\left(t\right)\left(µ+λ\right)=µp\_{1}\left(t\right)-µp\_{o}\left(t\right)-λp\_{o}\left(t\right)$$

$$δ\frac{p\_{o}\left(t\right)}{δt}=µp\_{1}\left(t\right)-λp\_{0}\left(t\right) (5) $$

Setting the differential equation to zero, equation (3.6) is obtained as;

$$µp\_{1}-λp\_{0}=0$$

$µp\_{1}=λp\_{0}$ So that,

$$p\_{1}=\frac{λ}{µ}p\_{o}=ρp\_{0 } (6)$$

We can find that,

1. server utilization factor,

$$ρ=\frac{λ}{µ} \left(7\right)$$

1. average number of passengers in the system,

$$l\_{s}=\frac{ρ}{1-ρ}=\frac{λ}{µ-λ} (8)$$

1. average number of queue,

$$l\_{q}=l\_{s}-ρ=\frac{ρ^{2}}{1-ρ} \left(9\right)$$

1. average time in system,

$$w\_{s}=\frac{1}{µ-λ} \left(10\right)$$

1. average time in queue,

$$w\_{q}=\frac{λ}{µ\left(µ-λ\right)}=\frac{ρ}{\left(µ-λ\right)} \left(11\right)$$

1. probability of having o passengers in the system (i.e. the service unit is idle)

$$p\_{0}=1-\frac{λ}{µ}=i-ρ \left(12\right)$$

1. probability of having n passengers in the system,

$$p\_{n}=ρ^{n}\left(1-ρ\right) (13)$$

From first principle,$y+δy-y$

The data displayed in the appendix was obtained by observational method.

From the appendix, it can be clearly noted that the time of arrival of each passenger, the time into service and the time out of service were recorded.

Analysis of the data was done using Queuing and Operation Research model toolpaks in Microsoft Excel Software package.

**Table 4.1: Frequency Of Passenger’s Arrival Within One Hour**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 8.00am-9.00am | 9.01am – 10.00am | 10.01am-11.00am | **TOTAL** |
| **Day 1** | 40 | 40 | 20 | 100 |
| **Day 2**  | 41 | 55 | 4 | 100 |
| **Day 3**  | 43 | 50 | 7 | 100 |
| **Day 4**  | 43 | 54 | 3 | 100 |
| **Day 5**  | 40 | 60 | - | 100 |
| **Day 6** | 43 | 55 | 2 | 100 |
| **Day 7**  | 50 | 50 | - | 100 |
| **Day 8**  | 47 | 53 | - | 100 |
| **Day 9**  | 46 | 54 | - | 100 |
| **Day 10**  | 45 | 55 | - | 100 |
| **TOTAL**  | 438 | 526 | 36 | 1000 |

From the data above, it can be deduced that passengers arrive mostly between 9.00 am to 10.00 am in the morning.

**Table 4.2: Frequency Of Passenger’s Arrival To Each Counter**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Counter A | Counter B | Counter C | Counter D | Counter E | Counter F |
| Day 1 | 19 | 16 | 20 | 16 | 17 | 12 |
| Day 2 | 23 | 15 | 16 | 15 | 16 | 15 |
| Day 3 | 18 | 17 | 13 | 23 | 16 | 13 |
| Day 4 | 16 | 18 | 19 | 19 | 16 | 12 |
| Day 5 | 16 | 20 | 17 | 21 | 13 | 13 |
| Day 6 | 18 | 15 | 18 | 20 | 14 | 15 |
| Day 7 | 19 | 21 | 19 | 17 | 11 | 13 |
| Day 8 | 17 | 21 | 18 | 16 | 16 | 12 |
| Day 9 | 20 | 18 | 18 | 17 | 14 | 13 |
| Day 10 | 19 | 19 | 20 | 18 | 11 | 13 |

From the data above, it can be deduced that passengers arrive mostly to be checked on counters A to D (which are economic traveling passengers) while few passengers (which are VIPs ) stay on counters E and F.

**Table 4.3: Arrival Duration and Service Duration For The 10days**

|  |  |  |
| --- | --- | --- |
|   | Arrival time (minutes) | Service time (minutes) |
| Day 1 | 144 | 194 |
| Day 2 | 121 | 187 |
| Day 3 | 119 | 200 |
| Day 4 | 123 | 197 |
| Day 5 | 116 | 194 |
| Day 6 | 111 | 186 |
| Day 7 | 119 | 197 |
| Day 8 | 110 | 188 |
| Day 9 | 106 | 191 |
| Day 10 | 110 | 184 |

**Table 4.4: Arrival Rate And Service Rate For The 10 days**

|  |  |  |
| --- | --- | --- |
|   | Arrival Rate (per minute)  | Service Rate (per minute) |
| Day 1 | 0.6944 | 0.5155 |
| Day 2 | 0.8264 | 0.5348 |
| Day 3 | 0.8403 | 0.5000 |
| Day 4 | 0.8130 | 0.5076 |
| Day 5 | 0.8621 | 0.5155 |
| Day 6 | 0.9009 | 0.5376 |
| Day 7 | 0.8403 | 0.5076 |
| Day 8 | 0.9091 | 0.5319 |
| Day 9 | 0.9434 | 0.5236 |
| Day 10 | 0.9091 | 0.5435 |

**Table 4.5: Mean Arrival Rate And Mean Service Rate**

|  |  |
| --- | --- |
| Mean Arrival Rate (λ) | 0.853912 per minute |
| Mean Service Rate (µ) | 0.52175 per minute |
| Number of Channels | 6 |

**Table 4.6: Queue Analysis**

|  |  |
| --- | --- |
| **Queue Station** | Que\_4 |
| Arrival Rate | 0.853912 |
| Service Rate/Channel | 0.52175 |
| Number of Servers | 6 |
| Max. Number in System | \*\*\* |
| Number in Population | \*\*\* |
| Type | M/M/6 |
| Mean Number at Station | 1.639309 |
| Mean Time at Station | 1.919763 |
| Mean Number in Queue | 0.002678 |
| Mean Time in Queue | 0.003137 |
| Mean Number in Service | 1.636631 |
| Mean Time in Service | 1.916627 |
| Throughput Rate | 0.853912 |
| Efficiency | 0.272772 |
| Probability All Servers Idle | 0.194551 |
| Prob. All Servers Busy | 0.007141 |
| Prob. System Full | 0 |
| Critical Wait Time | 1 |
| P(Wait >= Critical Wait) | 0.000733 |
| P(0) | 0.194551 |
| P(1) | 0.318408 |
| P(2) | 0.260558 |
| P(3) | 0.142146 |
| P(4) | 0.05816 |
| P(5) | 0.019037 |
| P(6) | 0.005193 |
| P(7) | 0.001416 |
| P(8) | 0.000386 |
| P(9) | 0.000105 |
| P(10) | 2.87E-05 |

From Table 4.6 above, the average number of passengers in the system, Ls is 1.6393 $ \~$ 2 passenger in the system at a time.

The average number of time a passenger spends in the system, Ws  is 1.919 $\~$ 2 minutes.

The average number of passengers in the queue, Lq is 0.002678.

The average number of time a passenger spends in the queue waiting for service, Wq is 1.916627 minutes $\~$ 2 minutes

The utilization factor, *ρ,* is 0.2727 i.e. each server is busy for 27% of the time

The probability that all servers are idle, P0 , i.e zero passengers on queue is 0.194551

The probability that all servers are busy is 0.007141

Table 4.5 above reveals that the probability that there are zero passenger on the queue is 0.194551 $\~$ 19%

The probability that there is one passenger on the queue is 0.318408 $\~$ 32%

The probability that there are two passengers on the queue is 0.260558 $\~$ 26%

The probability that there are three passengers on the queue is 0.142146 $\~$ 14%

The probability that there are four passengers on the queue is 0.05816 $\~$ 0.5%

The probability that there are five passengers on the queue is 0.019037$\~$ 0.19%

The probability that there are six passengers on the queue is 0.005193 $\~$ 0.052%

 *Figure 4.1: State probabilities*

The percentage of passengers in this system that waits for a threshold time of 2 minutes or less before they are attended to:

QTPMMS\_ServiceLevel (Threshold time, Arrival Rate, Service Rate, Servers)

QTPMMS\_ServiceLevel(2,0.853912,0.52175,6)= 0.9999= 99%

If 99% of the waits 2 minutes or less before they are attended to, it means that 1% waits longer.

With the same threshold time of 2 minutes, we obtain the number of servers needed to achieve a service level of 99% .

QTPMMS\_MinServers (Threshold\_time, Serviceslevel, Arrival Rate, Service Rate)

QTPMMS\_MinServers(2,99%,0.8539,0.5218) = 4

This implies that for 99% of the arriving passengers to spend a maximum of 2 minutes before being attended to, a minimum of 4 channels is sufficient.

*Fig 4.1: Graph showing the desired service levels with respect to the minimum number of the servers*

This study focused on examining the arrival rate, waiting time and service rate of passengers. In this research work, attention is focused on the application of queuing theory to the passengers boarding Qatar airways at Muritala Muhammed Airport, Lagos.

The average number of passengers in the system at a time is two.

The average number of time a passenger spends in the system is two minutes.

The average number of passengers in the queue is 0.002678.

The average number of time a passenger spends in the queue waiting for service is 2 minutes.

The utilization factor, *ρ* is 27% i.e. each server is busy for 27% of the time.

The probability that all servers are idle, P0, i.e. zero passengers on queue is 0.1946

The probability that all servers are busy is 0.0071

The probability that there are zero passenger on the queue is 19%

The probability that there is one passenger on the queue is 32%

The probability that there are two passengers on the queue is 26%

The probability that there are three passengers on the queue is 14%

The probability that there are four passengers on the queue is 0.5%

The probability that there are five passengers on the queue is 0.19%

The probability that there are six passengers on the queue is 0.052%

**CONCLUSION**

From the results obtained in this research work, it can be concluded that there exist no or less queue at the arrival counter on Qatar airway in Muritala Muhammed Airport, Lagos**.** Passengers are attended to almost immediately on their arrival. The maximum time a passenger spends on the system is few. The servers are efficient enough. For a more efficient service level, the number of servers could be increased.

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