**Open Neighbourhood Sombor degree based topological indices** **of basic graphs**

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**Abstract**

In this paper, we introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs such as Path, Cycle and Complete graphs.

**Keywords:** Open Neighbourhood, Degree based topological indices, Basic graphs.

**1. Introduction**

Let be a simple, finite and connected graph with vertices and edges. The degree of a vertex in a graph is denoted as . The first degree-based structure descriptors were conceived in the 1970s [3]. In 2019, S.Mondal et al.,[9,10] introduced the neighbourhood degree based topological indices. In 2021, V.Ravi et al.,[12] introduced some open neighbourhood degree based topological indices. In 1975, Milan Randić introduced the Randić index [8]. In 1972, Gutman and Trinajsti´c introduced the first and second Zagreb indices [1,2]. In 2021, I.Gutman[5] introduced the Sombor index. In 2021, V.R.Kulli [7] introduced the Banhatti-Sombor index. In 2024, I.Gutman et al.,[4] discussed the Elliptic Sombor index. In 2021, I.Gutman introduced the Reduced Sombor index. In 2024, I.Gutman[5] discussed the Euler Sombor index.

Motivated by the above studies, in this paper we introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs.

Now, we discuss the Open Neighbourhood Sombor degree based topological indices of aforesaid, where the open neighbourhood index is given by , represents the neighbourhood of vertex in the graph and denotes the degree of the vertex

* The **Open Neighbourhood Sombor Index** is defined as
* The **Open Neighbourhood Banhatti Sombor Index** is defined as
* The **Open Neighbourhood Elliptic Sombor Index** is defined as
* The **Open Neighbourhood Reduced Sombor Index** is defined as
* The **Open Neighbourhood of Euler Sombor Index** is defined as

Also, we proposed reciprocal of Open Neighbourhoods Degree Sum Based Sombor Indices are given below

* The **Reciprocal Open Neighbourhood Sombor Index** is defined as
* The **Reciprocal Open Neighbourhood Banhatti Sombor Index** is defined as
* The **Reciprocal Open Neighbourhood Elliptic Sombor Index** is defined as
* The **Reciprocal Open Neighbourhood Reduced Sombor Index** is defined as
* The **Reciprocal Open Neighbourhood of Euler Sombor Index** is defined as

**2. Main Results**

In this section we compute the Open Neighbourhood Sombor degree based topological indices of some basic graphs such as Path, Cycle and Complete graphs.

**2.1 Open Neighbourhood Sombor Degree Based Topological Indices of Path graph**

Let be the Path graph on vertices. The Open Neighbourhood Sombor index edge partitions for are Count

**Theorem:2.1.1**

Let be a path graph with vertices. Then

**Proof:**

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

**2.2 Open Neighbourhood Sombor Degree Based Topological Indices of Cycle graph**

Let be the Cycle graph on vertices. The Open Neighbourhood Somborindex edge partitions for are Count

**Theorem:2.2.1**

Let be a Cycle graph with vertices. Then

**Proof:**

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

**2.3 Open Neighbourhood Sombor Degree Based Topological Indices of Complete graph**

Let be the Cycle graph on vertices. The Open Neighbourhood Somborindex edge partitions for are Count .

**Theorem:2.3.1**

Let be a Complete graph with vertices. Then

**Proof:**

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

**3. Conclusion**

In this paper, we have introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs such as Path, Cycle and Complete graphs.

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