**GAUSSIAN ANTI-MAGIC LABELING FOR A GRAPH AND DIGRAPH**

*1S. Bala, 2T. Vijay, 3K. Thirusangu,*

*Department of Mathematics,*

*S.I.V.E.T College, Gowrivakkam, Chennai-73.*

*E-Mail:* [*1yesbala75@gmail.com*](mailto:1yesbala75@gmail.com) *,* [*2vijivijay31897@gmail.com*](mailto:2vijivijay31897@gmail.com)

***ABSTRACT***

*Graph labeling is one of the important research areas in graph theory. In this paper, we prove the existence of Gaussian anti-magic labeling for 4- regular graph of girth and Cayley digraph associated with 2- generator 2- group graph. Also we investigate the existence of and Fibonacci cordial labeling for the Cayley digraph associated with 2- generator 2- group graph.*

***Keywords:*** *Regular graph, Cayley digraph, 2-generator 2-group, Gaussian anti-magic labeling, Fibonacci Cordial labeling*

1. **INTRODUCTION AND DEFINITION**

The concept of graph labeling was introduced by Rosa [5] in 1967. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, magic labeling, anti-magic labeling, cordial labeling etc., have been studied in over 2500 papers [3]. In 1878, Cayley constructed a graph for a given group with a generating set which is now popularly known as Cayley graphs. A directed graph or digraph consists of a finite set of points called vertices and a set of directed arrows between the vertices.

Let be a finite group and be a generating subset of . The Cayley digraph denoted by, is the digraph whose vertices are the elements of , and there is an arc from whenever and . If then there is an arc from if and only if there is an arc from . The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [4]. For example, hypercube, butterfly, and cube-connected cycle’s networks are Cayley graphs [1].

In 2018, a new type of graph labeling called Gaussian Anti-magic labeling has been introduced by K. Thirusangu and A. Selvaganapathy [8], Gaussian anti-magic labeling in a graph is a function such that the induced function defined by for all , which results all the edge labels are distinct. A graph which admits Gaussian anti-magic labeling is called Gaussian anti-magic graph. Rokad and Ghodasara introduced a new labeling called Finonacci cordial labeling in 2016 [6], An Injective function , where is the Fibonacci number , is said to be Fibonacci cordial labeling if the induced function defined by satisfies the condition A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph.

A group is said to be a -group if . It is said to be   
2-generated if the minimal generating set of has exactly two elements. It is said to be a 2-group if Cayley digraph for the 2-generated 2-group has vertices and arcs with the vertex set of as and the arc set of as where and or where and . Denote the arcs in as and as or as and as . Clearly each vertex in has exactly two outgoing arcs out of which one arc is from the set and another is from the set .

A graph is said to be regular graph if degree of each vertex is equal. A graph is called -regular if degree of each vertex in the graph is . It is said to be 4-regular graph if degree of each vertex in the graph is 4. Girth of a graph G is defined as the length of smallest cycle. It is denoted by . The possible girth

4-regular graph with girth has the vertex set and edge set as and . It is denoted by (4-.

1. **MAIN RESULTS**

In this section, we prove the existence of the gaussian anti-magic labeling for 4-regular graph with girth and Cayley digraph associated with 2- generated 2- group. Also, we prove the existence of the Fibonacci cordial labeling for Cayley digraph associated with 2- generated 2- group.

**THEOREM 2.1:**

The graph (4- admits Gaussian anti-magic labeling.

**Proof:**

From the construction, we have a 4-regular graph with girth has vertices and edges.

Define the vertex set by

Vertex labelings are show in the table given below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Vertex** |  |  | **…** |  |  |
|  |  | … |  |  |

Define an induced function by

for all

Edge labelings obtained are shown in the table given below:

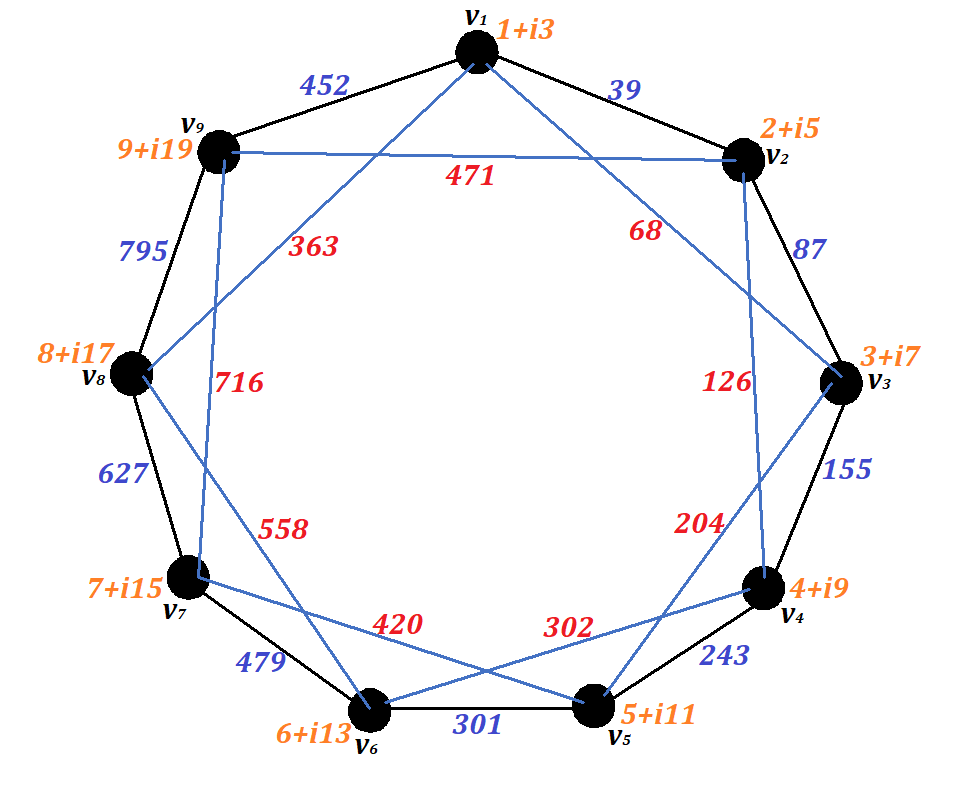
|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Edge E** |  |  | **…** |  |
| 39 | 87 | … |  |
|  |  | **…** |  |
|  |  | … |  |
|  |  | |  |
|  |  | |  |

Thus, all the edge labels are distinct.

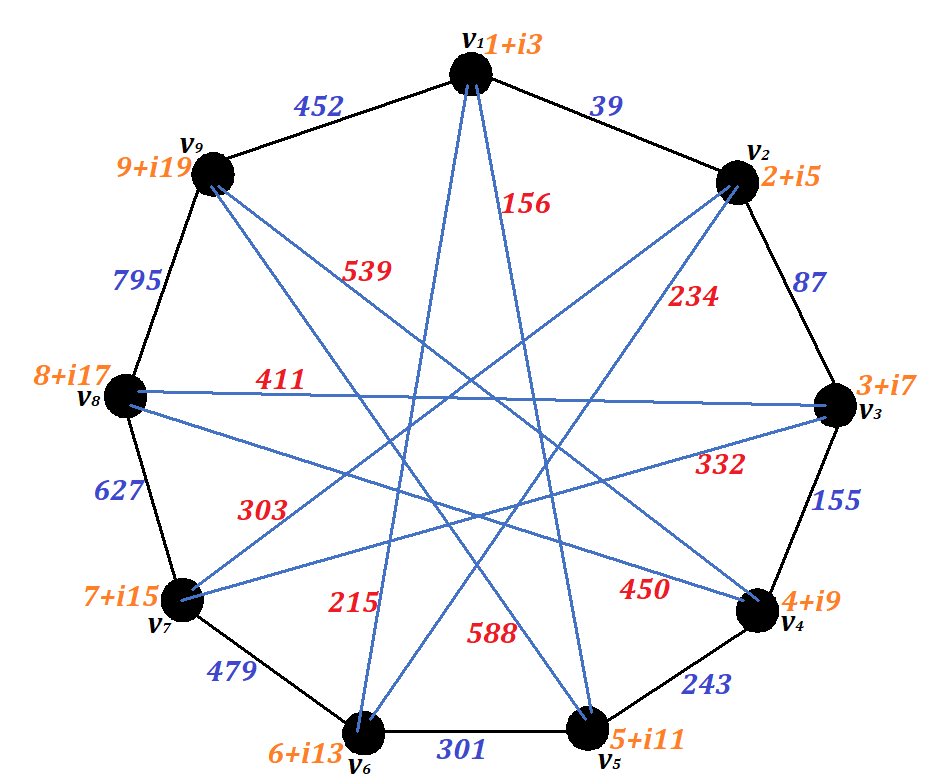
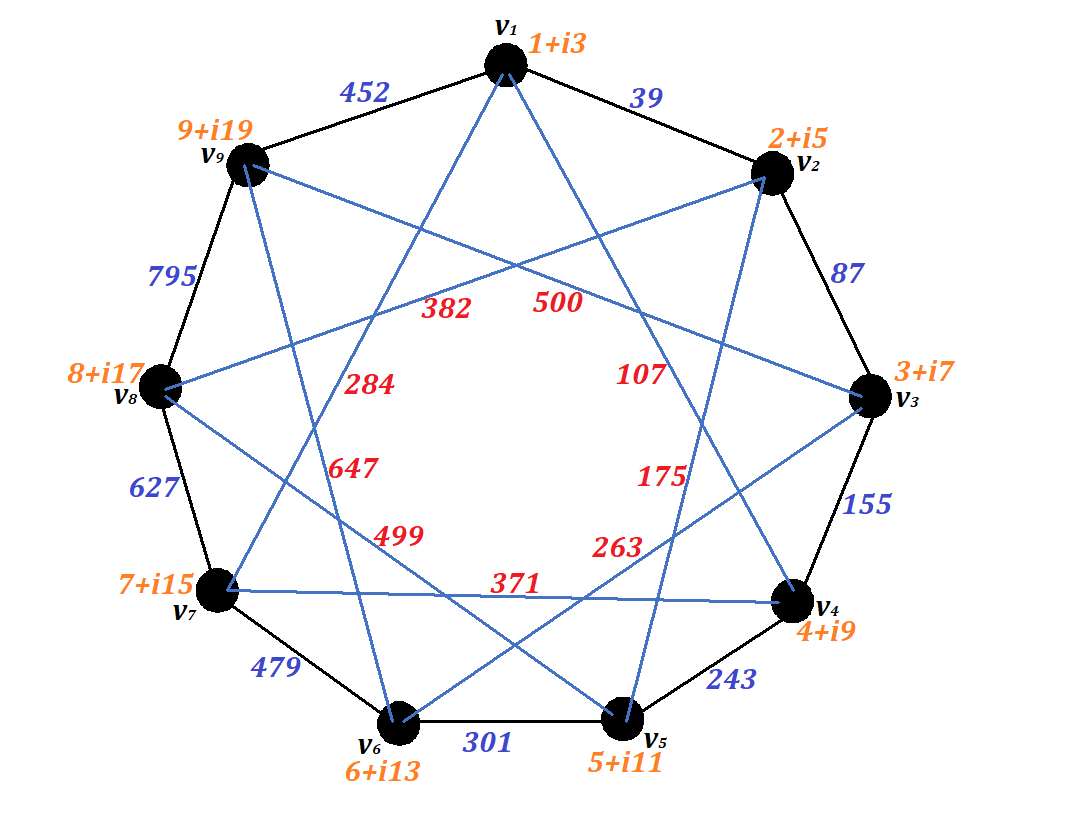
Therefore, the graph (4- admits Gaussian anti-magic labeling.

**EXAMPLE 2.1:**

Gaussian anti-magic labeling for graph (4-, graph (4-, graph (4- is given in Figure 2.1, Figure 2.2, Figure 2.3, respectively.



**Figure 2.1: Gaussian anti-magic labeling for graph (4-**



**Figure 2.2: Gaussian anti-magic Figure 2.3: Gaussian anti-magic**

**labeling for graph (4-. labeling for graph (4-.**

**THEOREM 2.2:**

The Cayley digraph associated with 2- generated 2- group admits Gaussian anti-magic labeling.

**Proof:**

From the structure of 2-generated 2-group with the generating set such that .

Let vertex set be represented as respectively. Clearly, Cayley digraph associated with 2-generated 2-group has 8 vertices and 16 edges.

Define a vertex set by

Vertex labelings are show in the table given below:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Vertex V** |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Define an induced function by

for all

Edge labelings obtained are shown in the table given below:

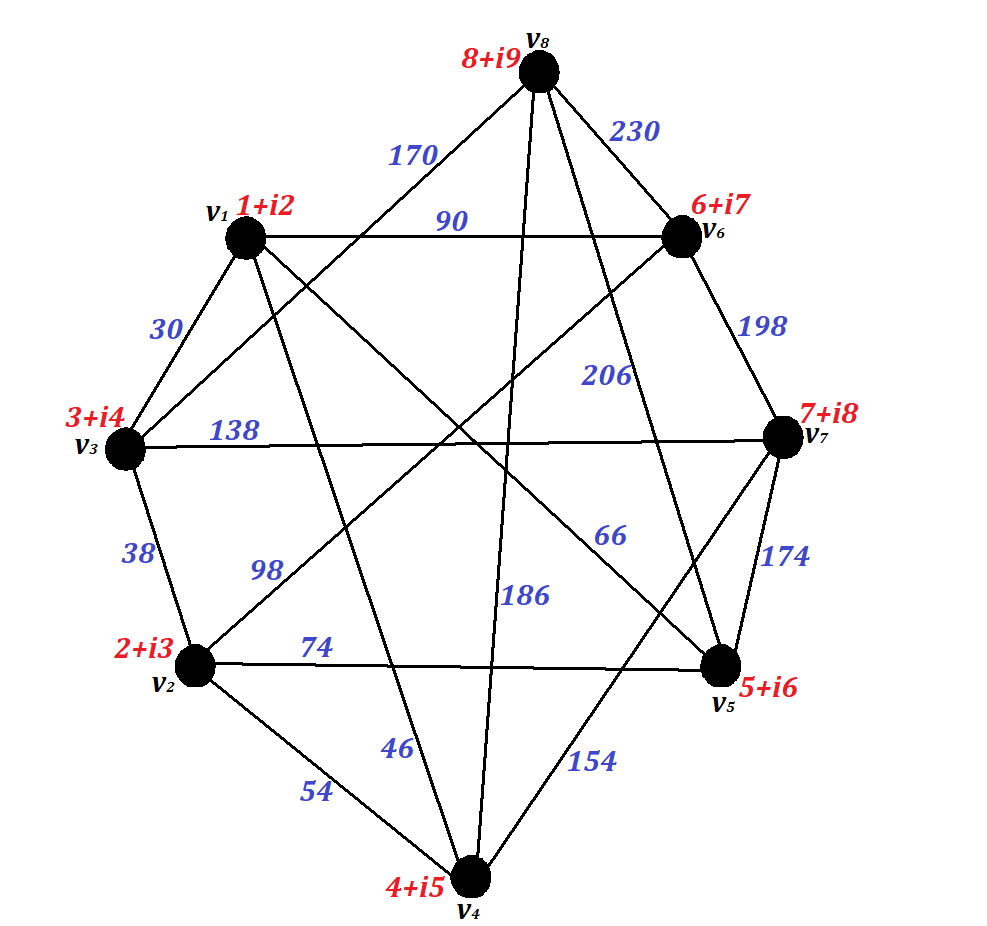
|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Edge E** |  |  |  |  |
| 30 | 46 | 66 | 90 |
|  |  |  |  |
| 38 | 54 | 74 | 98 |
|  |  |  |  |
| 138 | 170 | 154 | 186 |
|  |  |  |  |
| 174 | 206 | 198 | 230 |

Thus, all the edge labels are distinct.

Therefore, Cayley digraph associated with 2- generated 2- group admits Gaussian anti-magic labeling.

**EXAMPLE 2.2:**

Gaussian anti-magic labeling for Cayley digraph associated with 2-generated 2-group is given in Figure 2.4.



**Figure 2.4: Gaussian anti-magic labeling for**

**Cayley digraph associated with 2-generated 2-group.**

**THEOREM 2.3:**

The Cayley digraph associated with 2- generated 2- group admits Fibonacci cordial labeling.

**Proof:**

We know that, Cayley digraph associated with 2-generated 2-group has 8 vertices and 16 edges.

Define a vertex set by

.

Define an induced function by

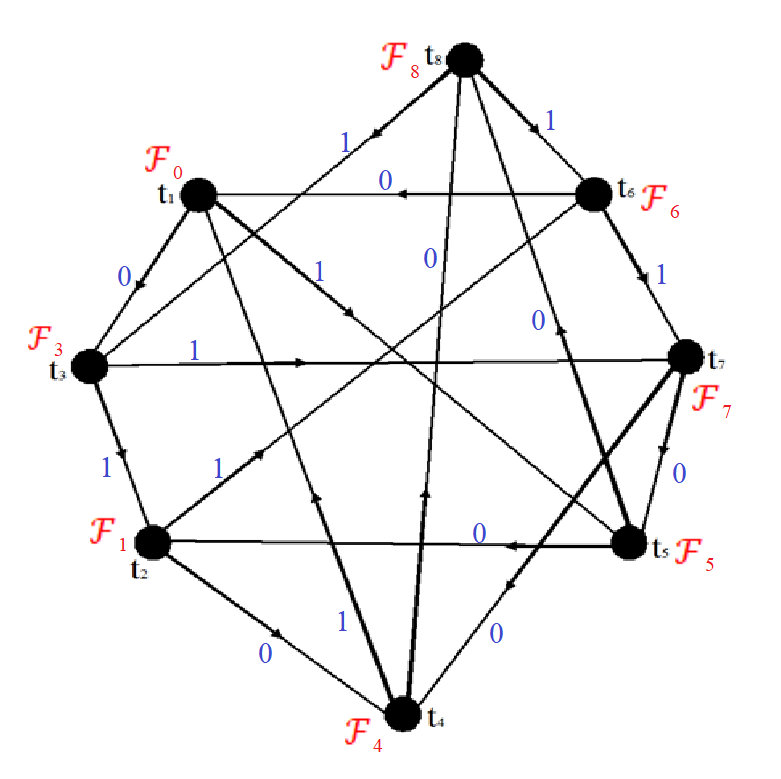
Thus, .

Clearly, the condition is satisfied.

Therefore, Cayley digraph associated with 2- generated 2- group admits Fibonacci cordial labeling.

**EXAMPLE 2.3:**

Fibonacci cordial labeling for Cayley digraph associated with 2-generated 2-group is given in Figure 2.5.



**Figure 2.5: Fibonacci cordial labeling for**

**Cayley digraph associated with 2-generated 2-group.**

**Conclusion:**

In this paper we have proved the existence of Gaussian anti-magic labeling for 4- regular graph of girth where and Cayley digraph associated with 2- generator 2- group graph. Also, we investigated the existence of Fibonacci cordial labeling for the Cayley digraph associated with 2- generator 2- group graph.

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