**PROPER D-LUCKY LABELING ON SOME EXTENDED DUPLICATE GRAPHS**

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**Abstract:**

In this paper we investigate the existence of Proper D-Lucky labeling on Extended Duplicate graphs.

Keywords: Graph Labeling, D-Lucky Labeling, Path graph

**1.INTRODUCTION:**

Esakkiammal et.al., [3] introduced the concept of **Proper D-lucky Labeling**. A d-lucky labeling is called proper if $l\left(u\right)\ne l\left(r\right)$ for every adjacent vertices $u$ and $r$. The proper d-lucky number of a graph is the least positive integ $k$ such that $G$ has a Proper d-lucky labeling with $\left\{1,2,….,k\right\}$ as the set of labels and is denoted by $η\_{pdl}\left(G\right)$.

**Definition: 2.1.2 (Extended Duplicate Graph of Path)**

Let $DG=\left(R\_{1},B\_{1}\right)$ be the Duplicate graph of a Path$ G(R,B$). The Extended Duplicate Graph of Path $EDG \left(P\_{m}\right)$ is obtained from the duplicate graph of path by joining $r\_{2}r\_{2}^{'},$ .Clearly it has $2m+2$ vertices and $2m+1$ edges,[6].

**Definition: 2.1.3 (Extended Duplicate Graph of Star)**

Let $DG(S\_{m})$ be the duplicate graph of Star. The Extended Duplicate Graph of Star $EDG\left(S\_{m}\right)$ is obtained from the duplicate graph of Star by joining $r\_{1}r\_{1}^{'}. $It has $6m$ vertices and $8m-3$ edges,[8].

**Definition: 2.1.4 (Extended Duplicate Graph of Twig)**

Let $DG(T\_{m})$ be the duplicate graph of Twig. The Extended Duplicate Graph of Twig $EDG\left(T\_{m}\right)$ is obtained from the duplicate graph of Twig by joining $r\_{2}r\_{2}^{'}.$ Cleary this $EDG \left(T\_{m}\right)$ has $6m+4$ vertices and $6m+3$ edges,[5].

**Definition: 2.1.5 (Extended Duplicate Graph of Bistar)**

Let $DG \left(BS\_{m,m}\right)$ be the duplicate graph of Bistar. The Extended Duplicate Graph of Bistar is obtained from the duplicate graph of Bistar by joining $r\_{1}r\_{1}^{'}. $Clearly $EDG \left(BS\_{m,m}\right)$ has $4m+4$ vertices and $4m+3$ edges,[7].

**2. MAIN RESULT**

In this section we investigate the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

**THEOREM: 2.1**

Extended Duplicate Graph of Path $EDG \left(P\_{m}\right)$ admits Proper d-lucky labeling with $η\_{dl}\left(EDG \left(P\_{m}\right) \right)=2$.

**Proof:**

From the structure of EDG$\left(P\_{m}\right)$, it is clear that EDG$\left(P\_{m}\right)$ has $2m+2$ vertices and $2m+1$ edges. To prove EDG $\left(P\_{m}\right)$ is d-lucky, define the function $l:R(G)\rightarrow N$ to label the vertices as follows:

For, $1\leq i\leq m$

1. $l \left(r\_{i}\right)=1$
2. $l (r\_{i}^{'})=2$

From the structure of the EDG$\left(P\_{m}\right)$, it is clear that the degrees of the vertices are as follows:

1. $d(r\_{1}) = d(r\_{i}^{'})= d(r\_{m+1}) = d(r\_{m+1}^{'})=1$
2. $d(r\_{2}) = d(r\_{2}^{'}) = 3$
3. $d(r\_{i}) = d(r\_{i}^{'}) = 3, for 3\leq i\leq m$
4. $c(r\_{1}$) = $c(r\_{m+1}$) = 3
5. $c(r\_{1}^{'}$) =$c(r\_{m+1}^{'}$) = 2
6. $c(r\_{2}$) = 9
7. $c(r\_{2}^{'}$) = 6
8. $c(r\_{i}$) = 9, for $3\leq i\leq m$
9. $c(r\_{i}^{'}$) = 4, for $3\leq i\leq m$

Clearly, $c(u)\ne c(v$) for any two adjacent vertices of EDG$\left(P\_{m}\right)$.Therefore EDG$\left(P\_{m}\right)$ admits d-lucky labeling with $η\_{dl}\left(EDG\left(P\_{m}\right)\right)=2$.

**EXAMPLE: 2.1**

Proper D-lucky labeling EDG$\left(P\_{5}\right)$ is shown in the figure 2.1.1 respectively.



**Figure 2.1.1**

**THEOREM 2.2**

Extended duplicate Graph of Star EDG$(S\_{m})$admits Proper d-lucky labeling with $η\_{dl}(ETG(S\_{m}))=2$.

**Proof:**

From the structure of EDG$(S\_{m})$, it is clear that EDG$(S\_{m})$ has 6m vertices and 8m-3 edges. To prove EDG$(S\_{m})$ is d-lucky, define the function $l:R(G)\rightarrow N$ to label the vertices as follows:

For, $1\leq i\leq m$

1. $l \left(r\_{i}\right)=1$
2. $l (r\_{i}^{'})=2$

From the structure of the EDG$\left(S\_{m}\right)$, it is clear that the degrees of the vertices are as follows:

1. $d(r\_{1})$ $= d(r\_{i}^{'})=m$ ,
2. $ d(r\_{i}) = d(r\_{i}^{'}) =1,for 2\leq i\leq m$
3. $c(r\_{1}$) $= 3m$,
4. $c(r\_{1}$) = 3, for $2\leq i\leq m$
5. $c(r\_{1}^{'}$) = $2m$
6. $c(r\_{1}^{'}$) = $2$, for $2\leq i\leq m$

Clearly, $c(u)\ne c(r)$ for any two adjacent vertices of EDG$\left(S\_{m}\right)$. Therefore EDG$\left(S\_{m}\right)$ admits d-lucky labeling with $η\_{dl}\left(EDG \left(S\_{m}\right)\right)=2$.

**EXAMPLE: 2.2**

Proper D-lucky labeling EDG $\left(S\_{5}\right)$ is shown in the figure 2.2.1 respectively.

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**Figure 2.2.1**

**THEOREM: 2.3**

Extended Duplicate Graph of Twig $EDG\left(T\_{m}\right)$ admits Proper d-lucky labeling with $η\_{dl}\left(EDG \left(T\_{m}\right) \right)=2$.

**Proof:**

From the structure of EDG$\left(T\_{m}\right)$, it is clear that EDG$\left(T\_{m}\right)$ has $6m+4$ vertices and $6m+3$ edges. To prove EDG$\left(T\_{m}\right)$ is d-lucky, define the function $l:R(G)\rightarrow N$ to label the vertices as follows:

For, $1\leq i\leq 3m+2$

1. $l \left(r\_{i}\right)=1$
2. $l (r\_{i}^{'})=2$

From the structure of the EDG$\left(T\_{m}\right)$, it is clear that the degrees of the vertices are as follows:

1. $d\left(r\_{1}\right)=1$,$d\left(r\_{2}\right)=5$
2. $d\left(r\_{3i}\right)=1,for 1\leq i\leq m$
3. $d\left(r\_{3i+1}\right)=1,for 1\leq i\leq m$
4. $d\left(r\_{3i+2}\right)=4,for 1\leq i\leq m-1$
5. $d\left(r\_{i}^{'}\right)=1$,$d\left(r\_{2}^{'}\right)=5$
6. $d\left(r\_{3i}^{'}\right)=1,for 1\leq i\leq m$
7. $d\left(r\_{3i+1}^{'}\right)=1,for 1\leq i\leq m$
8. $d\left(r\_{3i+2}^{'}\right)=4,for 1\leq i\leq m-1$
9. $d\left(r\_{3m+2}\right)=d\left(r\_{3m+2}^{'}\right)=1$
10. $c(r\_{1}$) = 3,$c(r\_{2}$) = 15
11. $c(r\_{1}^{'}$) = 2,$c(r\_{2}^{'}$) = 10
12. $c(r\_{3i}$) = $c(r\_{3i+1}$) = 3$,for 1\leq i\leq m$
13. $c(r\_{3i}^{'}$) = $c(r\_{3i+1}^{'}$) = $2,for 1\leq i\leq m$
14. $c(r\_{3i+2}$) = 12,$ c(r\_{3i+2}^{'}$) = $8, for 1\leq i\leq m-1$

Clearly, $c(u)\ne c(r)$ for any two adjacent vertices of EDG $\left(T\_{m}\right)$. Therefore EDG $\left(T\_{m}\right)$ admits d-lucky labeling with $η\_{dl}\left(EDG \left(T\_{m}\right)\right)=2$.

**EXAMPLE: 2.3**

Proper D-lucky labeling EDG$\left(T\_{3}\right)$ is shown in the figure 2.3.1 respectively.



**Figure 2.3.1**

**THEOREM: 2.4**

Extended Duplicate Graph of Bistar $EDG \left(BS\_{m,m}\right)$ admits Proper d-lucky labeling with $η\_{dl}\left(EDG \left(BS\_{m,m}\right) \right)=2$.

**Proof:**

From the structure of EDG $\left(BS\_{m,m}\right)$, it is clear that EDG $\left(BS\_{m,m}\right)$ has $4m+4$ vertices and $4m+3$ edges. To prove EDG $\left(BS\_{m,m}\right)$ is d-lucky, define the function $l:R(G)\rightarrow N$ to label the vertices as follows:

For, $1\leq i\leq 2m+2$

1. $l \left(r\_{i}\right)=1$
2. $l (r\_{i}^{'})=2$

From the structure of the EDG $\left(P\_{m}\right)$, it is clear that the degrees of the vertices are as follows:

1. $d\left(r\_{1}\right)=d\left(r\_{i}^{'}\right)=m+2$,
2. $d\left(r\_{i}\right)=d\left(r\_{i}^{'}\right)=1,for 2\leq i\leq m+1$
3. $d\left(r\_{m+2}\right)=d\left(r\_{m+2}^{'}\right)=m+1,for 1\leq i\leq m$
4. $d\left(r\_{m+2+i}\right)=d\left(r\_{m+2+i}^{'}\right)=1,for 1\leq i\leq m$
5. $c(r\_{1}$) = $3m+6$,$c(r\_{i}$)$=3,for 2\leq i\leq m+1$
6. $c(r\_{m+2}$) = $3m+3$
7. $c(r\_{m+2+i}$)$=3,for 1\leq i\leq m$
8. $c(r\_{1}^{'}$) = $2m+4$,$c(r\_{m+2}^{'}$) = $2m+2$
9. $c(r\_{i}^{'}$) = $2,for 2\leq i\leq m+1$
10. $c(r\_{m+2+i}^{'}$) = $2, for 1\leq i\leq m$

Clearly, $c(u)\ne c(r)$ for any two adjacent vertices of EDG $\left(BS\_{m,m}\right)$.Therefore EDG $\left(BS\_{m,m}\right)$ admits d-lucky labeling with $η\_{dl}\left(EDG \left(BS\_{m,m}\right)\right)=2$.

**EXAMPLE: 2.4**

Proper D-lucky labeling EDG $\left(BS\_{4,4}\right)$ is shown in the figure 2.4.1 respectively.

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**Figure 2.4.1**

**Conclusion:**

In this paper, we have confirmed the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

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