**PROPER D-LUCKY LABELING ON SOME EXTENDED DUPLICATE GRAPHS**

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**Abstract:**

In this paper we investigate the existence of Proper D-Lucky labeling on Extended Duplicate graphs.

Keywords: Graph Labeling, D-Lucky Labeling, Path graph

**1.INTRODUCTION:**

Esakkiammal et.al., [3] introduced the concept of **Proper D-lucky Labeling**. A d-lucky labeling is called proper if for every adjacent vertices and . The proper d-lucky number of a graph is the least positive integ such that has a Proper d-lucky labeling with as the set of labels and is denoted by .

**Definition: 2.1.2 (Extended Duplicate Graph of Path)**

Let be the Duplicate graph of a Path). The Extended Duplicate Graph of Path is obtained from the duplicate graph of path by joining .Clearly it has vertices and edges,[6].

**Definition: 2.1.3 (Extended Duplicate Graph of Star)**

Let be the duplicate graph of Star. The Extended Duplicate Graph of Star is obtained from the duplicate graph of Star by joining It has vertices and edges,[8].

**Definition: 2.1.4 (Extended Duplicate Graph of Twig)**

Let be the duplicate graph of Twig. The Extended Duplicate Graph of Twig is obtained from the duplicate graph of Twig by joining Cleary this has vertices and edges,[5].

**Definition: 2.1.5 (Extended Duplicate Graph of Bistar)**

Let be the duplicate graph of Bistar. The Extended Duplicate Graph of Bistar is obtained from the duplicate graph of Bistar by joining Clearly has vertices and edges,[7].

**2. MAIN RESULT**

In this section we investigate the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

**THEOREM: 2.1**

Extended Duplicate Graph of Path admits Proper d-lucky labeling with .

**Proof:**

From the structure of EDG, it is clear that EDG has vertices and edges. To prove EDG is d-lucky, define the function to label the vertices as follows:

For,

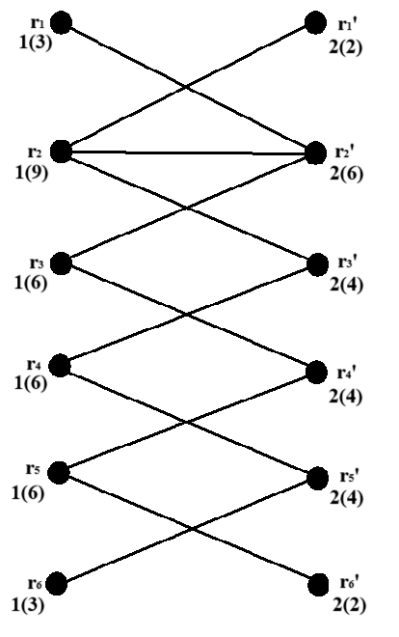
From the structure of the EDG, it is clear that the degrees of the vertices are as follows:

1. ) = ) = 3
2. ) =) = 2
3. ) = 9
4. ) = 6
5. ) = 9, for
6. ) = 4, for

Clearly, ) for any two adjacent vertices of EDG.Therefore EDG admits d-lucky labeling with .

**EXAMPLE: 2.1**

Proper D-lucky labeling EDG is shown in the figure 2.1.1 respectively.



**Figure 2.1.1**

**THEOREM 2.2**

Extended duplicate Graph of Star EDGadmits Proper d-lucky labeling with .

**Proof:**

From the structure of EDG, it is clear that EDG has 6m vertices and 8m-3 edges. To prove EDG is d-lucky, define the function to label the vertices as follows:

For,

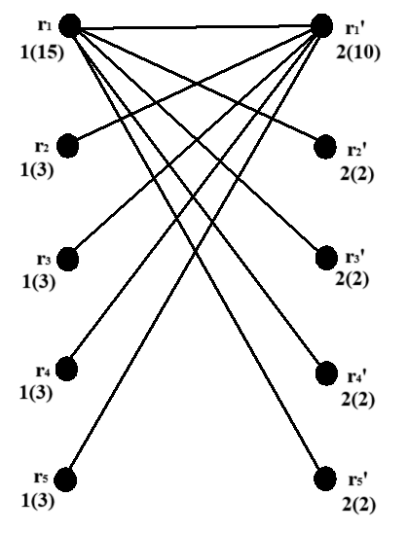
From the structure of the EDG, it is clear that the degrees of the vertices are as follows:

1. ,
2. ) ,
3. ) = 3, for
4. ) =
5. ) = , for

Clearly, for any two adjacent vertices of EDG. Therefore EDG admits d-lucky labeling with .

**EXAMPLE: 2.2**

Proper D-lucky labeling EDG is shown in the figure 2.2.1 respectively.

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**Figure 2.2.1**

**THEOREM: 2.3**

Extended Duplicate Graph of Twig admits Proper d-lucky labeling with .

**Proof:**

From the structure of EDG, it is clear that EDG has vertices and edges. To prove EDG is d-lucky, define the function to label the vertices as follows:

For,

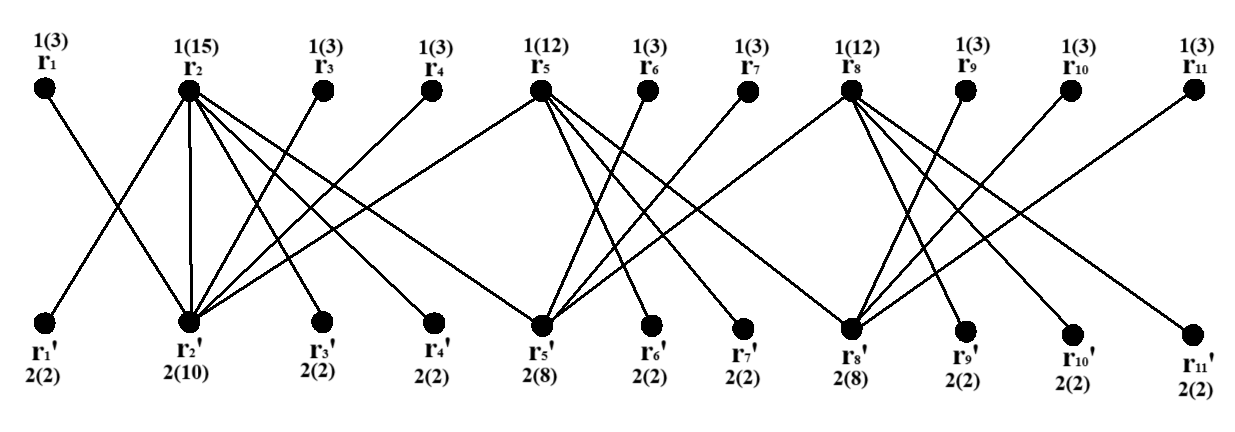
From the structure of the EDG, it is clear that the degrees of the vertices are as follows:

1. ,
2. ,
3. ) = 3,) = 15
4. ) = 2,) = 10
5. ) = ) = 3
6. ) = ) =
7. ) = 12,) =

Clearly, for any two adjacent vertices of EDG . Therefore EDG admits d-lucky labeling with .

**EXAMPLE: 2.3**

Proper D-lucky labeling EDG is shown in the figure 2.3.1 respectively.



**Figure 2.3.1**

**THEOREM: 2.4**

Extended Duplicate Graph of Bistar admits Proper d-lucky labeling with .

**Proof:**

From the structure of EDG , it is clear that EDG has  vertices and edges. To prove EDG is d-lucky, define the function to label the vertices as follows:

For,

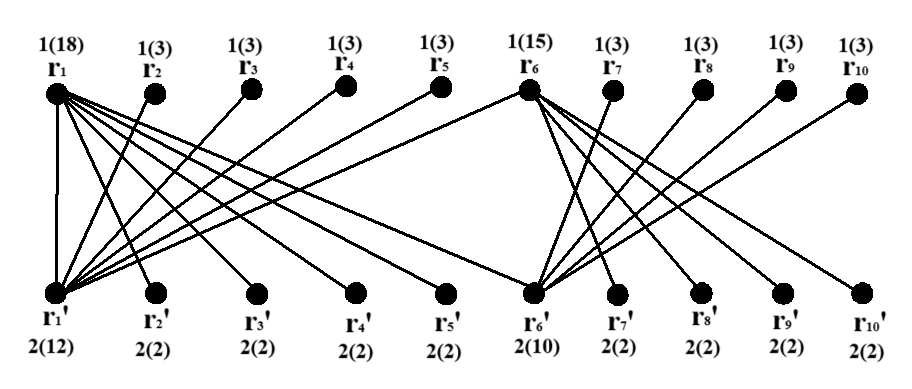
From the structure of the EDG , it is clear that the degrees of the vertices are as follows:

1. ,
2. ) = ,)
3. ) =
4. )
5. ) = ,) =
6. ) =
7. ) =

Clearly, for any two adjacent vertices of EDG .Therefore EDG admits d-lucky labeling with .

**EXAMPLE: 2.4**

Proper D-lucky labeling EDG is shown in the figure 2.4.1 respectively.

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**Figure 2.4.1**

**Conclusion:**

In this paper, we have confirmed the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

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