**“Unveiling Optical Rogue Waves: Exploring the Coherently Coupled Nonlinear Schrödinger Equation with Alternating Nonlinearities**

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**Abstract**

The research aims to examine optical rogue waves in the coherently coupled nonlinear Schrödinger equation with alternating signs of nonlinearities. The paper uses the Darboux transformation approach to investigate rogue wave patterns, such as single-peak and double-valley configurations, and dazzling rogue waves, which may have single or double peaks and no valleys in their paths. The paper observes two types of rogue waves: a dark wave with no troughs and a rogue wave with a brilliant peak and two valleys. The research in this paper can be helpful in various technologies, including fiber-based supercontinuum synthesis, laser systems, and Raman fiber amplifiers. Overall, the paper contributes to understanding optical rogue waves and their potential applications in various fields.

**Keywords;** Optical Rogue Waves, Nonlinear Schrödinger Equation, Alternating Nonlinearities, Coherently Coupled

**1. Introduction**

"Rogue waves," or waves that are much larger than the norm, have been reported in a wide variety of sectors, including nonlinear optics [1], water wave tanks [2], Bose-Einstein condensation [3], chaotic wave fields [4], and even the economics [5]. There has been a lot of attention from many different areas since the first empirical detection of optical rogue waves in 2007 [6]. Extensive study of these waves has been motivated by their potential use in a variety of technologies, including fiber-based supercontinuum synthesis [7], laser systems [8], and Raman fiber amplifiers [5, 6]. Rogue waves are more likely to develop with modulation instability [9]. Some examples of rogue wave morphologies are one-peak-two-valley profiles, two-peak-two-valley profiles, brilliant profiles, and dark profiles [10]. Many rogue waves have characterized the propagation of higher-order rogue waves, often arranged in a triangular, rhombus, or pentagonal pattern [11].

The nonlinear Schrödinger (NLS) equation has attracted much study because of its usefulness in these fields. Here, we look at the optical rogue waves described by the coherently coupled NLS equations with opposite signs of nonlinearities [12].

(1a)

(1b)

At which and Being complicated intensity that changes very gradually Represents the potency of discontinuity or its power. The derivatives about the direction of propagation and the delayed time are denoted by and respectively, correspondingly. In equation (2a), self-phase modulation exhibits a positive effect, while cross-phase modulation demonstrates a negative effect. Conversely, in equation (2b), the four-wave mixing term manifests a positive effect. Several studies have documented focusing-defocusing nonlinear Schrödinger (NLS) equations in various contexts, including artificial metamaterials, optical fibres, and magneto-optical waveguides [13]. The equations presented in reference [12]can be rephrased in an alternative manner.

(2a)

(2b)

**2. Darboux transformation for Eqs.**[**(2)**](https://www.sciencedirect.com/science/article/pii/S089396591830020X#fd2)

The Lax pair representation for equation (2) equations can be expressed as follows.

(3a)

(3b)

Where

For which is the 2 2 Eigen function of vectors, section array An additional serves as a symbol for the vector's transposition, with a focus on the spectroscopic variable. The equality requirement provides an accurate reduction to Eqs. (2).

Setting the stage for the shift , and putting it within Fractional Paired (3) yields

(4)

Where and use the identical structures as U and V. unless, of course, the possibilities replaced by and Matrix of Darboux dimensions entails or means . And It is a lax pair with having two distinct sets of values.

And Are lax combinations having multiple sets of equations (3) with. The DT The solution to Eq. (2) May be written as

(5)

With

(6)

The Updated answers as well as are derived as

(7)

At which

## 3. Rogue-wave solutions for Eqs. [(2)](https://www.sciencedirect.com/science/article/pii/S089396591830020X#fd2)

While using the Darboux transformation (DT) with zero seed solutions will help you acquire multi-soliton solutions [13], using the DT with plane-wave solutions as the starting solutions will make it easier to generate rogue-wave solutions [14]. This part considers two possible beginning solutions: a plane wave and zero.

(8)

Whereas the actual average is. We observed the Eigen function responses to Lax Pair (3), which are

(9a)

(9b)

(9c)

(9d)

Using,, In the appearance of complex parameters, By substituting the Eigen function Results (9) into Equations (7), we may derive the rogue-wave responses to Eq. (2).

1. When a = 0, then results will be obtained by the germinating ideas. and
2. When and Reference [15]examines the Akhmediev breathers and rogue-wave solutions to Eqs. (2).
3. When and More generally applicable rogue-wave solutions to Eqs. (2) Will be addressed below.

## 4. Conclusions

The paper explores optical rogue waves in the coherently coupled nonlinear Schrödinger equations with alternating signs of nonlinearities. The Darboux transformation approach investigates rogue wave patterns, such as single-peak and double-valley configurations, and dazzling rogue waves, which may have single or double peaks and no valleys in their paths. The paper observes two types of rogue waves: a dark wave with no troughs and a rogue wave with a brilliant peak and two valleys. The paper shows that a sizeable rogue wave without troughs can eventually split into two separate peaks while maintaining its shape.

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