# ON APPROXIMATION BY SOME INTEGRAL MODIFICATION OF JAKIMOVSKI-LEVIATAN OPERATORS

NANDITA GUPTA

Government Polytechnic, Mahasamund Chhattisgarh, **India.**

Email: nandita dec@yahoo.com

**Abstract**. In the present paper we propose an integral modification of Jakimovski- Leviatan operators involving Appell Polynomials. Using Korovkin Theorem we obtain the approximation properties of these operators. We compute an estimate

of the order of approximation of a continuous functionsby means of the operator via first order modulus of continuity. We also give an asymptotic

estimate through Voronovskaja - type result for these operators.

***2020 AMS Subject Classification***. Primary 41A25, 41A36.

***Key Words and Phrases***. Linear positive operators, Appell Polynomials, Order of approximation, Modulus of continuity, Voronovskaja - type theorem .

# Introduction

Jakimovski and Leviatan [[3]](#_bookmark21) introduced a new type of operators *Pn* by using Appell polynomials as follows,



for *f* ∈ *E*[0*,* ∞).B. Wood proved in [[6]](#_bookmark22) that the operators *Pn* are positive on [0*,* ∞)

*an*

if and only if ≥ 0*, n* ∈ N

*g*(1)

Recall that Appell polynomials are polynomials defined as follws: Let

 be an analytic function in the disc |*u*| *< r,* (*r >* 1) and



be the Appell polynomials defined by the identity



Ibrahim Bu¨yu¨kyazici et. al. [[2](#_bookmark19)] gave a chlodowsky type generalization of Jakimovski - Leviatan operators given by

 

with *bn* a positive increasing sequence with the properties



Motivated by the operators given in (1.2) and ([1.3)](#_bookmark1) we consider the following integral modification of the operator (1.3):



 where *bn* is a positive increasing sequence satisfying [(1.4](#_bookmark2)).We see by construction of the operator *An* that the condition for positivity given by Wood for operator *Pn* is applicable here also.So throughout this paper we will assume that the operators *An* are positive.

1. **Approximation properties of** *An*(*f* ; *x*)

We denote by *CE*[0*,* ∞) the set of all continuous functions f on [0*,* ∞) with the property that |*f* (*x*)| ≤ *βeαx*for all *x* ≥ 0 and some positive finite *α* and *β*.

Following lemma was given in [[3](#_bookmark21)].

**Lemma 2.1.** *From* (1.2) *we have*

 *,*





Using lemma [(2.1](#_bookmark4)) and eq. ([1.5)](#_bookmark3) we get following results.

**Lemma 2.2.** *The operators An defined by eq.* (1.5) *satisfy,*



**

*where ei*(*t*) = *ti, i* = 0*,* 1*,* 2*,* 3*,* 4*.*

*An*(*e*4; *x*) = *x*

+

+

*Proof.* From the definition of *An*(*f* ; *x*) and lemma [(2.1](#_bookmark4)) , we have





 



For e3 we have



 

and

 

 

 

 The following result can be easily derived from the previous lemma.

**Lemma 2.3.** *The operators An defined by eq.* (1.5) *satisfy,*



**Theorem 2.4.** *For f* ∈ *CE*[0*,* ∞)*, the operators An converge uniformly to f on*

[0*, a*]*, a >* 0 *as n* ∈ N*.*

*Proof.* From lemma [(2.2](#_bookmark5)) we have

lim *An*(*ei*; *x*) = *ei*(*x*)*, i* = 0*,* 1*,* 2*.*

*n*→∞

On applying Korovkin theorem [[1]](#_bookmark20) we get the desired result.

# Order Of Approximation

In this section we give an estimate of the order of approximation of a function *f* ∈ *CE*[0*,* ∞) by means of the operator *An* , using the first order modulus of conti- nuity.

Let *f* ∈ *C*[0*, b*].The modulus of continuity of f denoted by *ω*(*f, δ*),is defined to be



The modulus of continuity of the function f in C[0,b] gives the maximum oscillation of f in any interval of length not exceeding *δ >* 0.

 It is well known that for any *δ >* 0 and each *s* ∈ [0*, b*]



**Theorem 3.1.** *Forf* ∈ *CE*[0*,* ∞) *and x* ∈ [0*, a*]*, a >* 0 *we have*



*Proof.* By using eq. (3.1) linearity of operators *An* we obtain



Using the Cauchy - Schwarz inequality we obtain



 

 

From eq. [(2.2](#_bookmark7)) ,for *x* ∈ [0*, a*]*, a >* 0 we obtain



Taking and using eq. [(3.3](#_bookmark12)) in eq. (3.2) we get the desired result.

**4. Theorem of Voronovskaja - type**

Now we give a Voronovskaja - type relation for the operator *An*.

**Theorem 4.1.** *If f* ∈ *C*2 [0*,* ∞) *then*

*E*



*uniformly for x* ∈ [0*, a*]*, a >* 0*.*

*Proof.* For a fixed *x*0 ∈ [0*,* ∞),by Taylor’s formula we have for every *t* ∈ [0*,* ∞)



 where *ψ*(*t*; *x*0) is a function belonging to the space *CE*[0*,* ∞) and lim*t*→*x*0 *ψ*(*t*; *x*0) = 0.Then by (4.1) and lemma (2.2) we can write for every *n* ∈ N,



From eqs. ([2.1](#_bookmark6)) and ([2.3)](#_bookmark8) we have



By Cauchy-Schwarz inequality we get for *n* ∈ N



From [(2.3](#_bookmark8)) we have



 Let *ϕ*(*t, x*0) = *ψ*2(*t*; *x*0)*, t* ≥ 0.Then *ϕ*(*t, x*0) ∈ *CE*[0*,* ∞) and

 Then from Theorem (2.4) we have

lim *An*(*ψ*2(*t*; *x*0); *x*0) = lim *An*(*ϕ*(*t*; *x*0); *x*0) = *ϕ*(*x*0; *x*0) = 0 (4.7)

*n*→∞ *n*→∞

uniformly with respect to *x*0 ∈ [0*, a*]. So by eqs. [(4.6](#_bookmark17))-([4.7)](#_bookmark18) we have



then, taking the limit as lim*n*→∞ in [(4.2](#_bookmark14)) and using [(4.2](#_bookmark14)),(4.4) and (4.5) we have



Hence the theorem.

**References**

1. F. Altomare, M. Campiti, Korovkin Type Approximation Theory and its Applications, in: De Gruyter Studies in Mathematics, vol. 17, Walter de Gruyter, Berlin, New York, 1994.
2. Ibrahim Bu¨yu¨kyazici ,Hande Tanberkan , Sevilay Krc Serenbay , C¸ iˆgdem Atakut,Approximation by Chlodowsky type Jakimovski - Leviatan operators , Journal of Computational and Applied Mathematics 259 (2014) 153-163.
3. A. Jakimovski, D. Leviatan, Generalized Szsz operators for the approximation in the infinite interval, Mathematica (Cluj) 34 (1969) 97103.
4. A.D. Gadjiev, On P. P. Korovkin type theorems, Mat. Zametki 20 (1976) 781786; Transl. in Math. Notes (56) (1978) 995 - 998.
5. A.D. Gadjiev, The convergence problem for a sequence of positive linear operators on bounded sets and theorems analogous to that of P.P. Korovkin, Dokl. Akad. Nauk SSSR 218 (5) (1974); Transl. in Soviet Math. Dokl. 15 (5) (1974) 1433 - 1436.
6. B. Wood, Generalized Szsz operators for approximation in the complex domain, SIAM J. Appl. Math. 17 (4) (1969) 790 - 801.