**VAGUE FEEBLY CLOSED SETS & VAGUE FEEBLY OPEN SETS**

**IN VAGUE TOPOLOGICAL SPACES**

**Mary Tency E.L**

**Assistant Professor, Department of Mathematics**

**Sri Ramakrishna College of Arts & Science, Coimbatore - 06**

**Mail.Id:** **tencyelawrence@gmail.com**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**ABSTRACT**

This paper studies the concepts of a new class of Vague Closed sets & Vague Feebly Open sets in vague topological space also some basic properties and the key theorems of these classes were discussed here.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**KEYWORDS**

Vague set , Vague topology , Vague closed set , Vague open set .

**1.0 INTRODUCTION**

 The theory of vague sets was first initiated by Gau and Buehrer [1] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. Maheswari and Jain (1978) [4], Ibraheem (2008) [2,3] introduced feebly open and feebly closed sets, feebly generalized closed (briefly g-closed) sets, generalized feebly closed (briefly g -closed) sets respectively. In 2017,Vigneshwaran.M and Velmeenal. M [5], studied on RG - closed sets in topological spaces.

 In this article, we introduce the concept of Vague feebly open sets and Vague feebly closed sets in VTS. We also analyzed their characterizations and investigated their properties with suitable examples. For a subset of a VTS , vague feebly closure of , vague feebly interior of and the vague complement of are denoted by respectively.

**1.1 VAGUE FEEBLY OPEN AND VAGUE FEEBLY CLOSED SETS**

**Definition 1.1.1:** Let and be any two vague subsets of a VTS. Then is vague q-neighbourhood with B if there exists a VOS with qOB. If A is not vague quasi-coincident with B then we write qB.

Thus qB if and only if for each , . i.e.,.

**Proposition 1.1.2:** Let be a VTS. Then for a VS of a VTS X, is the union of all vague points such that every vague semi open set O with qO is vague q-coincident with .

**Proof:** Let .

Suppose there is a vague semi - open set ‘O’ such that qO and A.

 Oc A, where Oc is vague semi - closed.

Also, Oc and Oc

. This is a contradiction to our assumption.

Therefore, for every vague semi - open set ‘O’ with qO is vague q-coincident with .

Conversely, for every vague semi - open set ‘O’ with qO is vague q-coincident with . Suppose . Then there is a vague semi - closed set G A withG. Hence V(Gc ) is a vague semi - open set with q(Gc) and qA. i.e., A(x) (Gc)c = G. This is a contradiction to our assumption. Therefore, .

**Proposition 1.1.3:** Let be a VTS. Let and are two vague subsets of a VTS. Then

1. qB A Bc.
2. If A B = then qB
3. qB, for each qA.

**Proof: (**i) Proof follows from the definition 1.1.1

(ii) Let Then min

 and (or) and

(i.e =

.

Hence qB. This proves (ii).

(iii) Let and qA. Then )

Also implies that Bc.

i.e., B.

Therefore, qB. Thus each qA, qB.

Suppose,. Then does not implies

This is a contradiction to our assumption.

Therefore . This proves (iii).

**Proposition 1.1.4:** Let be a VTS. Let A be a vague subset of a VTS . Then

(i) = and

 =

1. = and = .

**Proof:** (i) It is true that.

Since is vague open, ,

 =

From the above, we have

. This proves (i).

(ii) It is true that, = and

By this (ii) is proved.

**Proposition: 1.1.5:** Let be a VTS. Let A be a vague subset of a VTS . Then .

**Proof:** Let .

Then by using the proposition 1.1.2, .

 This implies that .

i.e., .

**Theorem 1.1.6:** Let be a VTS. If a vague subset is vague open,

 then .

**Proof:** By using the above proposition 1.1.5, we have .

Therefore it is sufficient to prove .

Let . Then c.

By using proposition 1.1.2, ).

By using proposition 1.1.4, =

This can be written as .

Also, is vague semi - open. By using proposition 4.1.3, we have

Therefore.

**Theorem 1.1.7:** Let be a VTS. If a vague subset is vague closed,

then .

**Proof:** If A is vague closed, then V(Ac ) is vague open.

By theorem 1.1.6,

.

Then by .

Taking complement on both sides, we get .

**Definition 1.1.8:** A subset in a VTS is called Vague feebly open in if there exists an VOS U such that . The complement of is a .

**Proposition 1.1.9:** A vague subset A of a VTS is

if and only if .

**Proof:** If is , then by the definition 1.1.8, we have , where U is a VOS. Then by theorem 1.1.6,

Since U is vague open, we have

it follows that

.

Thus, .

Assume that . Now, .

. Take .

Then U is a VOS in X, such that .

By theorem 1.1.6, .

Therefore, A is .

**Theorem 1.1.10:** Let be a VTS. A set is said to be a

if and only if .

**Proof :** Follows from proposition 1.1.5 and proposition 1.1.9.

The following example is an example of .

**Example 1.1.11:** Let where

Let . Here .

Hence A is a .

**Definition 4.1.12:** A vague subset of a VTS is a if and if .

**Proposition 1.1.13:** Every VOS is a .

Proof: Let be a VOS in X.

Therefore and .

Now, .

.

Hence A is a vague feebly open set.

The converse of the above proposition is not true as shown in the example below.

 **Example 1.1.14:** Let

where then be a VTS.

Let .

Here is not a VOS since .

But .

Hence, .

Therefore, A is .

**Proposition 1.1.15:** A vague subset in a VTS is a if and only

 if it is vague semi - open and vague pre - open.

**Proof:** Let be a in X.

Then

.

Hence A is a vague semi - open set.

Since is in X, we have

.

Hence A is vague pre - open set.

Conversely, let is a vague semi - open set,

Therefore so that .

Hence, .

Since A is a vague pre - open set, and

hence .

Then by proposition 1.1.9**,** A is .

**Definition 1.1.16:** Let be a VTS and .

1. The intersection of all Vague feebly closed subsets of the space containing A is called the Vague feebly closure of and denoted by and also

1. The union of all Vague feebly open subsets of the space contained in is called Vague feebly interior of A and is denoted by

It is known that

**Proposition 1.1.17 :** If and are two then is a .

**Proof:** If and are two , then by proposition 1.1.9**,**

 and .

Now U .

Since ,

 U

Also, U

This implies .

Hence AUB is a .

**Proposition 1.1.18:** Arbitrary union of vague feebly open sets is a vague feebly open set.

**Proof:** Let{Ai} be a collection of of a VTS .

Then there exists a VOS i such that

Hence i is a .

**Example 1.1.19:** Intersection of any two s need not be a as shown in the example below.

Let be a vague topology on X.

where

. Let and be s in

but is not a in .

**Proposition 1.1.20:** The vague closure of a VOS is a .

**Proof:**  Let be a in X.

Take ,

Now, .

Since .

.

.

Hence A is a .

**Proposition 1.1.21:** Let A be a in the VTS and suppose B.

Then B is a .

**Proof:** Let A be a in the VTS and

B be any vague subset of X such that B.

Since A is , there exists a VOS U such that A.

Since B and and thus B B.

Hence B is .

**Definition 1.1.22:** A vague subset of is said to be a vague feebly generalised closed set (VGCS in short) if whenever and

**Definition 1.1.23:** A vague subset of is said to be a vague closed set (V in short) if whenever and

**Definition 1.1.24:** A vague subset A of a VTS is if there is a VCS U in X such that

**Proposition 1.1.25:** A vague subset A of a VTS is if and only if .

**Proof:** If A is then by the definition 1.1.24

there is a VCS U such that . Also

Since U is a VCS, .

Therefore .

Hence

Conversely, Assume that .

Since , Take .

Then U is a VCS in X such that

By the definition 1.1.24, A is a .

**Proposition 1.1.26:** A vague subset is a if and only if V( is a .

**Proof:** Let A be a .

Then by the proposition 1.1.2, .

Taking compliment on both sides .

This implies .

 Hence is a .

Conversely, let is a , then .

Taking complement on both sides, .

Then .

Therefore .

Hence A is a .

**Theorem 1.1.27:** A vague subset A is a if and only if .

**Proof:** Let A be a . Then is vague feebly open.

 By using theorem 1.1.10 .

Taking complement on both sides . . Therefore is a . By proposition 1.1.26. A is a .

The following is an example of .

**Example 1.1.28:** Let where

then be a VTS and let .

.

Therefore, is a .

**Proposition 1.1.29:** Every VCS is .

**Proof:** Let be a VCS in X. Then

Since

By proposition 1.1.25, A is a .

The converse of the above theorem need not be true as shown in the example below

**Example 1.1.30:** Let

where then be a vague topological space and

let .

Here is a .

**Proposition 1.1.31:** If and are any two s, then and

.

**Proof:** By theorem 1.1.27, .

This implies. T

his implies .

Hence is a .

**Proposition 1.1.32:** Finite intersection of is a .

**Proof:** Let{Ai} be a collection of of a VTS .

Then by the definition 1.1.24 there exists a VCS Vi such that

 for each i.

Now

Hence Ai is a .

**Remark 1.1.32:** Union of any two s need not be a as shown in the example.

**Example 1.1.33:** Let be a VT on X,

where

 and

 let a VSand be two in but is not a in .

**2. Conclusion**

This article has delved into the intricate concepts of vague feebly closed sets and vague feebly open sets within the framework of vague topological spaces. Through a rigorous exploration of these mathematical constructs, we have unveiled their properties and relationships and characteristics of vague topology. As we wrap up this study, it becomes evident that the investigation of vague feebly closed and open sets opens avenues for further research and exploration. We propose the following areas for future work: Extension to Higher Dimensions, Relation to Other Topological Concepts, Applications in Real-world Problems and Generalization to Different Vague Topological Spaces.

**3. References**

1. **Gau. W.L. and Buehrer. D.J**, Vague sets, IEEE Transactions on Systems, Man, and Cybernetics, 23, 1993, 610–614.
2. **Ibraheem. D,** On feebly generalized closed sets, Uruk Refereed Journal for Scientific Studies Issued by Al-Muthana University (1), 2008.
3. **Ibraheem. D,** On generalized feebly closed sets, Uruk Refereed Journal for Scientific Studies Issued by Al-Muthana University (1), 2008, 131 – 139.
4. **Maheshwari. S.N and Jain. P.G,** Some new mappings, Mathematica, 24 (47), 1982, 53 – 55.
5. **Vigneshwaran. M and Velmeenal. M,** On RFG-Closed Sets in Topological Spaces, Journal of Informatics and Mathematical Sciences, 9(3), 2017, 799–814.