VAGUE 𝜸\* GENERALIZED CLOSED SETS

**Priyamalini A M1, Francina Shalini A2**

1PG Student,Department of Mathematics, Nirmala college for women , Coimbatore ,Tamil Nadu, India

2Assistant Professor, Department of Mathematics, Nirmala college for women, Coimbatore,Tamil Nadu,India

**ABSTRACT**

The purpose of this paper is to explore the concept of vague sets in order . In this paper, we introduce the notion of vague 𝛾\* generalized closed sets and investigate some of their properties. Vague sets are an extension of fuzzy sets. In the context of the theory of vague 𝛾\* generalized closed sets, we construct few examples that are useful and we also obtain some interesting theorems.

Keywords: Vague sets, vague topology, vague 𝛾\* generalized closed sets.

1. **INTRODUCTION**

The concept of fuzzy sets and its operation was introduced by Zadeh [8] in 1965.After that Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. The concept of vague set was introduced by Gau and Burherer[4]. In this paper, the concept of vague 𝛾\* generalized closed set had been introduced and some of its properties have been discussed.

1. **PRELIMINARIES**

**Definition 2.1 [3]:**

 A vague set A in the universe of discourse X is characterized by two membership functions given by:

* A true membership function and
* A false membership function

 where is a lower bound on the grade of membership of derived from the “evidence for ”, is a upper bound on the negation of derived from the “evidence for ”, and . Thus the grade of membership of any element in the vague set A is bounded by a subinterval of .

 If the actual grade of membership of then .

 The vague set A is written as where the interval is called the “vague value” of in A, denoted by .

**Definition 2.2 [3]:**

 Let A and B be VSs of the form and . Then

* if and only if and for all
* if and only if and

**Definition 2.4[4]:**

A vague topology (VT in short) on X is a family of VSs in X satisfying the following axioms.

* ,for any
* for any family{

In this case the pair is called a Vague topological space (VTS in short) and anyVS in is known as a Vague open set (VOS in short) in X.

The complement of a VOS A in a VTS is called a vague closed set (VCS in short ) in X.

**Definition 2.5[5]:**

A vague set 𝐴 = {(𝑥: [(𝑥), 1 − 𝑓(𝑥)]|𝑥 ∈ 𝑋 )} in a VTS (𝑋, 𝜏) is said to

Vague 𝛾\* closed set (V 𝛾\*CS) if ((𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴)) ⊆ 𝐴

* Vague 𝛾\* closed set (V 𝛾\*CS) if (𝑉𝑖𝑛(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴)) ⊆ 𝐴
* Vague 𝛾\* open set (V 𝛾\* OS) if 𝐴 ⊆ (𝑉𝑐𝑙(𝐴)) 𝖴 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴))

 **Definition 2.6[6]:**

 Let 𝐴 be a Vague set in (𝑋, 𝜏), then

* 𝛾\*(𝐴) ⊆ 𝐴 ∩ (𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))

**Definition 2.7[4]:**

 A Vague Set 𝐴 = {(𝑥: [(𝑥), 1 − 𝑓𝐴 (𝑥)]|𝑥 ∈ 𝑋 )} in a VTS (𝑋, 𝜏)is said to be a

* Vague semi closed set (VSCS) if ((𝐴)) ⊆ 𝐴.
* Vague α-open set (V𝛼OS) if 𝐴 ⊆ 𝑉𝑖𝑛𝑡 (((𝐴))).
* Vague regular closed set (VRCS) if 𝐴 = (𝑉𝑖𝑛(𝐴)).
* Vague regular open set (VROS) if 𝐴 = ((𝐴)).
* Vague pre-open set (VPOS) if 𝐴⊆ ((𝐴)).
* Vague pre-closed set (VPCS) if (𝑉𝑖𝑛𝑡 (𝐴)) ⊆ 𝐴

 **VAGUE GENERALIZED CLOSED SETS**

**Definition 3.1:**

A vague set 𝐴 in a VTS (𝑋, 𝜏) is said to be a vague 𝛾\* generalized closed sets (V 𝛾\* GCS) if (𝑉𝑖𝑛(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))⊆ U whenever 𝐴 ⊆ 𝑈 and 𝑈 is a V𝛾\* OS in (𝑋, 𝜏)**.**

**Example 3.2:**

 Let 𝑋 = {𝑎, 𝑏} and 𝜏 = {0, 1, G} is VTS in 𝑋, where = {𝑥: [0.5,0.4]; [0.5,0.6]} .Then (𝑋, 𝜏) is VTS. Let

𝐴 = {𝑥:[0.4,0.4];[0.6,0.6]} is VTS in (𝑋, 𝜏) .We have 𝐴 ⊆ 1 .Now 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))=0 ⊆ 1, Where 1 is VOS in X. This implies that A is a V 𝛾\* GCS in 𝑋.

**Theorem 3.3:**

 Every VCS is V 𝛾\* GCS in (𝑋, 𝜏) but not conversely in general.

**Proof:**

Let 𝐴 be a VCS in (𝑋, 𝜏). Let 𝐴 ⊆ 𝑈 where 𝑈 is VOS in (𝑋, 𝜏). Since A is a VCS . As (𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))⊆ . Hence 𝐴 is V 𝛾\*GCS in (𝑋, 𝜏).

**Example 3.4:**

 In Example 3.2,the VTS 𝐴 = {𝑥:[0.4,0.4];[0.6,0.6]} is V 𝛾\* GCS 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴)) but not an VCS in (𝑋, 𝜏) , as 𝑉𝑐𝑙(𝐴)

**Theorem 3.5:**

 Every VSCS is V 𝛾\*GCS in (𝑋, 𝜏) but not conversely.

 **Proof**:

Let A be a VSCS and let 𝐴 ⊆ U and be a VOS in (X, τ). Since A is every VSCS 𝑉𝑖𝑛t(𝑉𝑐𝑙(𝐴)) ⊆ 𝐴.

Now 𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))⊆ 𝐴 ⊆ 𝑈 .Hence A is a V 𝛾\*GCS in (X, τ).

**Example 3.6:**

In Example 3.2, the vague set 𝐴 = {𝑥:[0.4,0.4];[0.6,0.6]} is a V 𝛾\* GCS 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴)) but not VSCS in (X, τ) as 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡(𝐴)) = 1 𝐴.

**Theorem 3.7:**

 Every VRCS is V 𝛾\* GCS in (𝑋, 𝜏) but not conversely.

**Proof:**

Let A be a VRCS in (X, τ). Since every VRCS is VCS by theorem 3.3, A is a V 𝛾\*GCS in (X, τ).

**Example 3.8:**

 In Example 3.2, the vague set 𝐴 = {𝑥:[0.4, 0.4];[0.6,0.6]} be a VTS in (X, τ). Then A is a V 𝛾\* GCS as whenever but not VRCS in (X, τ) as

**Theorem 3.9**:

 Every VCS is V 𝛾\* GCS in (𝑋, 𝜏) but not conversely in general.

**Proof:**

Let 𝐴 be a V𝛼CS in (𝑋, 𝜏) . Let 𝐴⊆𝑈 where 𝑈 be a in (𝑋, 𝜏) . Now (𝑉𝑖nt(𝐴)) ∩ 𝑉𝑖𝑛𝑡(𝑉𝑐𝑙(𝐴))⊆ 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡 (𝑉𝑐𝑙 (𝐴)) 𝑉𝑐𝑙(𝑉𝑖𝑛𝑡 (𝑉𝑐𝑙 (𝐴)) ⊆ 𝐴 , by hypothesis. Hence 𝐴 is V 𝛾\* GCS in (𝑋, 𝜏).

**Example 3.10:**

In Example 3.2, the vague set 𝐴 = {𝑥:[0.4, 0.4];[0.6,0.6]} be a VTS in (X, τ). Then A is a V 𝛾\* GCS as whenever but not VCS in (X, τ) as

**Theorem 3.11:**

 Every VCS is V 𝛾\* GCS in (𝑋, 𝜏) but not conversely in general.

**Proof:**

Let 𝐴 be a VCS in (𝑋, 𝜏). Let 𝐴⊆𝑈 where 𝑈 be a in (𝑋, 𝜏) .Since A is a ,Hence 𝐴 is V 𝛾\* GCS in (𝑋, 𝜏).

 **Example 3.12:**

  Let 𝑋 = {𝑎, 𝑏} and 𝜏 = {0, 1, G} is VTS in 𝑋, where = {𝑥: [0.5,0.4]; [0.5,0.6]} .Then (𝑋, 𝜏) is VTS. Let = {𝑥:[0.4, 0.4];[0.6,0.6]} is a V 𝛾\* GCS as whenever but not VCS in (X, τ) as

1. **CONCLUSION**

In this present paper we introduced and studied a concept of vague generalized closed sets. The basic properties of vague generalized closed sets are also presented and discussed.

1. **REFERENCES**

[1] Atanassov. K., Intuitionistic fuzzy Set, Fuzzy set and systems, 20 (1986), 87-96.

[2] Coker, D., An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88(1997), 81-89.

[3] Coker, D., and Demirci, M., 1995, “On Intuitionistic Fuzzy Points”, Notes on intuitionistic fuzzy sets 1, 79-84.

[4] Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.

[5] Hanafy, I. M., “Intuitionistic Fuzzy γ-Continuity”, 2009, Canad. Math. Bull. 52 544-554

[6] Kanimozhi, R., and Jayanthi, D**.**, 2016, “On intuitionistic fuzzy generalized γ closed sets”, International journal of Scientific Engineering and Applied Science, 2, 1-5.

[7] Riya V. M., and Jayanthi, D., “ Intuitionistic fuzzy γ\* generalizd closed sets”, Advanced in fuzzy mathematics,(2017),389-410.

[8] Zadeh . L. A., Fuzzy Sets, Information and control, 8 (1965), 338-353.

.