**VAGUE** *α* **GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES**

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**ABSTRACT**

In this paper a new concept of vague closed sets called vague *α* generalized closed sets is introduced . We have constructed some examples which are useful in the theory of Vague generalized closed sets in topological spaces.

**Keywords:** Vague topology , vague *α* generalized closed sets , vague *α* generalized open sets.

**1.INTRODUCTION**

The concept of fuzzy sets was introduced by Zadeh in 1965.On the otherhand Gau and Buehrer.D.J in 1993 introduced an extension of fuzzy set theory and basic concepts of vague set theory. Biswas in 2006 gave various results in vague set. The theory of fuzzy topology was introduced by Chang in 1968. Mariapresenti and Arockia Rani in 2016, defined the vague generalized alpha closed set in topological spaces. Closed sets are fundamental objects in a topological space. One can define the topology on a set by using either the axioms for the closed sets. In 1970, Levine initiated the study of generalized closed sets. By definition, a subset S of a topological space X is called generalized closed if cl (A)⊆U whenever A ⊆U and U is open. This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets.

**2.PRELIMINARIES**

A vague set A in the universe of discourse X is characterized by two membership functions given by:

* + 1. A true membership function *tA* : *X* → [0*,* 1] and
    2. A false membership function *fA* : *X* → [0*,* 1],

where *tA*(x) is lower bound on the grade of membership of x derived from the “evidence for x”. *fA*(x) is a lower bound on the negation of x derived from the “evidence against x” and *tA*(x) + *fA*(x) ≤ 1. Thus the grade of membership of x

in the vague set A is bounded by a sub interval [ *tA*(x),1 – *fA*(x)] of [0, 1]. This

indicates that if the actual grade of membership *µ*(x), then *tA*(x) ≤ *µ*(x) ≤ *fA*(x). The vague set A is written as A = {*< x,* [*tA*(*x*)*,* 1–*fA*(*x*)] *>*| *x* ∈ *X*} ,

where the interval [*tA*(x) ,1 – *fA*(x)] is called the ”vague value of x” in A and is denoted by *VA*(x).

**2.1 Definition**

Let A and B be two vague sets of the form

A = {*< x,* [*tA*(*x*)*,* 1–*fA*(*x*)] *>*| *x* ∈ *X*} and B = {*< x,* [*tB*(*x*)*,* 1–*fB*(*x*)] *>*| *x* ∈

*X*}.Then,

a) A ∩ B = {*< x,* [*tA*(*x*) ∧ *tB*(*x*))*,* ((1 − *fA*(*x*) ∧ 1 − *fB*(*x*)))] *> /x* ∈ *X*}

b) A ∪ B = {*< x,* [*tA*(*x*) ∨ *tB*(*x*))*,* ((1 − *fA*(*x*) ∨ 1 − *fB*(*x*)))] *> /x* ∈ *X*}

c) A = B if and only if A ⊆ B and B⊆ A

d) *Ac* = {*< x,* [*tA*(*x*)*,* 1–*fA*(*x*)] *>*| *x* ∈ *X*}

e) A ⊆ B if and only if *tA* ≤ *tB* and 1 - *fA*(x) ≤ 1-*fB*(x)

We shall use the notation A ={*< x,* [*tA*(*x*)*,* 1–*fA*(*x*)] *>*} instead of {*< x,* [*tA*(*x*)*,* 1–*fA*(*x*)] *>*| *x* ∈ *X*} .

**2.2 Definition** A vague topology (VT in short) on X is a family *τ* of vague sets (VS in short) in X satisfying the following axioms.

1. 0,1 ∈*τ*

2. *G*1 ∩ *G*2 ∈ *τ*

3. ∪ *Gi* ∈ *τ* for any family { G*i* / ∈ J }⊆ *τ*

In this case the pair (X, *τ* ) is called vague topological space (VTS in short) and vague set in *τ* is known as vague open set (VOS in short) in X. The complement *Ac* of vague open set in vague topological space is (X, *τ* ) called vague closed set (VCS in short) in X.

**3.VAGUE** *α* **GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES**

### Definition 3.1

A vague set A in a vague topological space X is said to be a vague *α* Generalized closed set (V*α*gCS) if V*α*Cl(A) ⊆ U whenever A ⊆ U and U is a vague open set (VOS) in X. The complement C(A) of a V*α*gCS A is a V*α*gOS in X .

### Theorem 3.2

Let (X, *τ* ) be a vague topological space A ∈ V*α*gC(X) and foe every B ∈ VS(X) , A⊆ B ⊆ V*α* Cl(A) implies B∈ V*α*gC(X) .

*Proof.*

Let a vague set B ⊆ U and U be a vague open set in (X, *τ* ) . Then since A ⊆ B , A ⊆ U and A is a V*α*gCS ,V*α*cl(A) .

By hypothesis , B ⊆ V*α* cl(A).

Therefore V*α* cl(B) ⊆ V*α*cl(V*α* cl(A)) ⊆ U , Therefore,V*α* cl(B) ⊆ U .

Hence B is a V*α*gCS(X) .

A vague set A in a vague topological space X is said to be a vague *α* Generalized closed set (V*α*gCS) if V*α*Cl(A) ⊆ U whenever A ⊆ U and U is a vague open set (VOS) in X. The complement C(A) of a V*α*gCS A is a V*α*gOS in X .

### Theorem 3.3

The union of any two V*α*gCSs is a V*α*gCS in a vague topological space X.

*Proof.*

Let A and B be any two V*α*gCSs in a vague topological space X. Let A ⊆ B ⊆ U where U is a vague open set in X.

Then A ⊆ U and B ⊆ U.

Now V*α*cl(A ⊆ B) = (A ⊆ B) ⊆ Vcl(VInt(Vcl(A ⊆ B))) ⊆ (A ⊆ B) ⊆ Vcl(Vcl(A

⊆ B)) ⊆ (A ⊆ B) ⊆ Vcl(A ⊆ B) ⊆ Vcl(A ⊆ B) = Vcl(A) ⊆ Vcl(B) ⊆ U ⊆ U = U, by hypothesis. Hence A ⊆ B is a V*α*gCS in X.

### Theorem 3.4

Every Vague alpha closed set A is a V*α*gCS in X but not converse *.*

*Proof.*

Let A ⊆ U where U is a Vague open set in X .

Now V*α*cl(A) = A ⊆ Vcl(Vint(Vcl(A))) ⊆ A ⊆ A = A ⊆ Uby hypothesis. Therefore , A is a V*α*gCS in X .

### Theorem 3.5

Every Vague *α* closed set A is V*α*CS in X but not converse .

*Proof.*

Let A ⊆ U , U is a Vague open set in X .

Then , V*α*Cl(A) = A ⊆ Vcl(Vint(Vcl(A))) ⊆ A ⊆ A = A ⊆ U , by hypothesis. Hence A is a V*α*CS in X .

### Theorem 3.6

If A is a vague open set and a V*α*gCS in (X,*τ* ). Then A is a V*α*-CS in (X, *τ* ).

*Proof.*

Since A ⊆ A and

A is a Vague open set in (X, *τ* ). By hypothesis, V*α*cl(A) ⊆ A.

But A ⊆ V*α*Cl(A). Therefore V*α*Cl(A) = A.

Hence A is a V*α*-CS in (X,*τ* ) .

**4. CONCLUSION**

This dissertation is a study of Vague *α* generalized closed sets in topological spaces. The Vague *α* generalized closed sets and Vague *α* generalized open sets in topological spaces and further defined some examples related to open and closed sets .

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