##### FEKETE SZEGO INEQUALITY FOR A CLASS OF ANALYTIC FUNCTIONS APPROACHING TO CLASS OF CONVEX FUNCTIONS IN THE LIMIT FORM AND CLASS OF STARLIKE FUNCTIONS DIRECTLY

By **Gurmeet Singh**

 Khalsa College Patiala-147001, Punjab, India

 Email:meetgur111@gmail.com

 **Abstract**

We introduce a class of analytic functions and obtain sharp upper bounds of the functional for the analytic function belonging to this class with special character that it tends to the class of convex functions as .

**Keywords:**Univalent functions, Starlike functions, Close to convex functions and bounded functions

**2020 Mathematical Sciences Classification: 30C50**

## 1 Introduction

Let denote the class of functions of the form

 (1.1)

which are analytic in the unit disc . Let be the class of functions of the form (1.1), which are analytic univalent in .

 In 1916, Bieber Bach [1, 2] proved that for the functions . In 1923, Löwner [10] proved that for the functions .

 With the known estimates and , it was expected to try to find some relation between and for the class **,** Fekete and Szegö [4] used Löwner’s method to prove the following well known result for the class .

Let , then

 (1.2)

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes [3, 9].

Let us define some subclasses of .

We denote by , the class of univalent starlike functions

and satisfying the condition

 (1.3)

We denote by , the class of univalent convex functions

and satisfying the condition

 (1.4)

A function is said to be close to convex if there exists such that

 (1.5)

The class of close to convex functions is denoted by C and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

 (1.6)

 (1.7)

It is obvious that is a subclass of and is a subclass of .

We introduce a new subclass as

and we shall denote this class as

We shall deal with two subclasses of defined as follows in our next paper:

 (1.8)

 (1.9)

 Symbol stands for subordination, which we define as follows:

**Principle of Subordination*.*** *Let and be two functions analytic in . Then is called subordinate to F(z) in if there exists a function analytic in satisfying the conditions and such that and we write*

By , we denote the class of analytic bounded functions of the form

 (1.10)

 It is known that

 (1.11)

## 2 Preliminary Lemmas.

 For , we write so that

 (2.1)

## 3 Main Results

**Theorem 3.1**. Let

*The results are sharp.*

*Proof***.** By definition of , we have

 (3.4)

 Expanding the series (3.1), we get

 (3.5)

Identifying terms in 3.2, we get

 (3.6)

 (3.7)

From (3.3) and (3.4), we obtain

 (3.8)

 Taking absolute value and using Triangular inequality, (3.5) can be rewritten as

 (3.9)

Using (1.9) in (3.6), simple calculations yield

 (3.10)

***Case* I.** . In this case, (3.10) can be rewritten as

 (3.11)

***Subcase* I (a).**

Using (1.9), (3.8) becomes

 (3.12)

***Subcase* I** **(b).**

We obtain from (3.8)

 (3.13)

***Case* II*.***

 Preceding as in case I, we get

 (3.14)

***Subcase* II** **(a).**

(3.11) takes the form

 (3.15)

Combining subcase I (b) and subcase II (a), we obtain

 (3.16)

***Subcase* II (b).**

Preceding as in subcase I (a), we get

 (3.17)

Combining (3.9), (3.13) and (3.14), the theorem is proved.

Extremal function for (3.1) and (3.3) is defined by

where

and

Extremal function for (3.2) is defined by ,

where and .

**Corollary 3.2.** *Putting and applying limit as in the theorem, we get*

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent convex functions.

**Corollary 3.3.** *Putting in the theorem, we get*

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

**References**

1. Alexander, J.W *Function which map the interior of unit* *circle upon simple regions*, Ann. Of Math., **17** (1995),12-22.
2. Bieberbach, L. Uber die Koeffizientem derjenigen Potenzreihen, welche eine schlichte Abbildung des Einheitskrsises vermitteln , *S. – B . Preuss. Akad. Wiss.* **38** (1916), 940-955.
3. De Branges L.*,* A proof of Bieberbach Conjecture, *Acta. Math*., **154** (1985),137-152.
4. Duren, P.L., Coefficient of univalent functions, Bull*. Amer. Math. Soc.*, **83** (1977), 891-911.
5. Fekete, M. and Szegö, G, Eine Bemerkung uber ungerade schlichte funktionen, *J.* *London Math. Soc*., 8 (1933), 85-89.
6. Garabedian, P.R. And Schiffer, M., AProof for the Bieberbach Conjecture forthe fourth coefficient, *Arch. Rational Mech. Anal*., 4 (1955), 427-465.
7. Kaur, C. and Singh, G., Approach To Coefficient Inequality For A New Subclass Of Starlike Functions With Extremals , *International Journal Of Research In Advent Technology,* **5**(2017) *,*
8. Kaur, C. and Singh, G., Coefficient Problem For A New Subclass Of Analytic Functions Using Subordination , *International Journal Of Research In Advent Technology,* **5**(2017) *,*
9. Keogh, F.R. and Merkes , E.P., A coefficient inequality for certain classes of analytic functions *, Proc. Of Amer. Math. Soc.*, **20** (1989), 8-12.
10. Koebe, P., Uber Die uniformisiesrung beliebiger analyischeer Kurven*, Nach. Ges. Wiss.* *Gottingen* (1907), 633-669.
11. Lindelof ,E., Memoire sur certaines inegalities dans la theorie des functions monogenes et sur quelques proprieties nouvellles de ces fontions dans la voisinage d′un point singulier essential, *Acta Soc. Sci. Fenn*., **23** (1909) , 481-519.
12. Ma ,W. and Minda , D. unified treatment of some special classes of univalent functions , *In Proceedings of the Conference on Complex Analysis ,*Z. Li, F. Ren , I. Yang and S.Zhang (Eds),Int. Press Tianjin (1994) , 157-169.
13. Miller, S.S., Mocanu, P.T. And Reade, M.O., All convex functions are univalent and starlike, *Proc. of Amer. Math. Soc.*, 37 (1973), 553-554.
14. Nehari, Z. (1952), *Conformal Mappings*, Mc Graw- Hill, New York.
15. Nevanlinna, R., Uber die Eigenshaften einer analytischen funkion in der umgebung einer singularen stele order Linte*, Acta Soc. Sci. Fenn*., **50** (1922) , 1-46.
16. Pederson, R., A proof for the Bieberbach conjecture for the sixth coefficient*,* *Arch. Rational Mech. Anal*., 31 (1968-69), 331-351.
17. Pederson, R. and Schiffer, M., A proof for the Bieberbach conjecture for the fifth coefficient, *Arch. Rational Mech. Anal.*, 45 (1972), 161-193.
18. Rani, M., Singh, G., Some Classes Of Schwarzian Functions And Its Coefficient Inequality That Is Sharp, *Turk. Jour. Of Computer and Mathematics Education*, **11** (2020), 1366-1372.
19. Rathore, G. S., Singh , G. and Kumawat, L. et.al., Some Subclaases Of A New Class Of Analytic Functions under Fekete-Szego Inequality, *International Journal Of Research In Advent Technology,* **7**(2019) *,*
20. Rathore, G. S., Singh , G., Fekete – Szego Inequality for certain subclasses of analytic functions ,  *Journal Of Chemical , Biological And Physical Sciences,* **5**(2015) *,*
21. Singh , G, Fekete – Szego Inequality for a new class and its certain subclasses of analytic functions , *General Mathematical Notes*, **21** (2014),
22. Singh , G, Fekete – Szego Inequality for a new class of analytic functions and its subclass, *Mathematical* *Sciences International Research Journal*, 3 (2014),
23. Singh, G., Construction of Coefficient Inequality For a new Subclass of Class of Starlike Analytic Functions, *Russian Journal of Mathematical Research Series*, **1** (2015), 9-13.
24. Singh, G., Introduction of a new class of analytic functions with its Fekete – Szegö Inequality, *International Journal of Mathematical Archive,* **5** (2014), 30-35.
25. Singh, G, An Inequality Of Second and Third Coefficients For A Subclass Of Starlike Functions Constrcted Using Nth Derivative, *Kaav International* *Journal Of Science, Engineering And Technology*, **4** (2017), 206-210.
26. Singh , G, Fekete – Szego Inequality for asymptotic subclasses of family of analytic functions, *Stochastic Modelling And Applications*, 26 (2022),
27. Singh , G, Coefficient Inequality For Close To Starlike Functions Constrcted Using Inverse Starlike Classes , *Kaav International* *Journal Of Science, Engineering And Technology* , **4** (2017) , 177-182.
28. Singh , G, Coefficient Inequality For A Subclass Of Starlike Functions That Is Constrcted Using Nth Derivative Of The Functions In The Class , *Kaav International* *Journal Of Science, Engineering And Technology* , **4** (2017) , 199-202.
29. Singh , G, Singh, Gagan, Fekete – Szegӧ Inequality For Subclasses Of A New Class Of Analytic Functions , *Proceedings Of The World Congress On Engineering* , (2014) , .
30. Singh , G, Sarao , M. S. , and Mehrok , B. S., Fekete – Szegӧ Inequality For A New Class Of Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013) , .
31. Singh , G, Singh, Gagan, Sarao , M. S. , Fekete – Szegӧ Inequality For A New Class Of Convex Starlike Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013) , .
32. Singh , G, Singh, P.,Fekete – Szegӧ Inequality For Functions Belonging To A Certain Class Of Analytic Functions Introduced Using Linear Combination Of Variational Powers Of Starlike And Convex Functions, *Journal Of Positive School Psychology* , **6** (2022) , 8387-8391.
33. Singh, G. , Fekete – Szegӧ Inequality For Functions Approaching to A Class In The Limit Form and another Class directly, *Journal Of Information And Computational Sciences*, .
34. Singh, G. and Kaur, G., Coefficient Inequality for a Subclass of Starlike Function generated by symmetric points, *Ganita*, **70** (2020), 17-24.
35. Singh ,G. and Kaur, G., Coefficient Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) *,*
36. Singh ,G. and Kaur, G., Fekete-Szegӧ Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) *,*
37. Singh ,G. and Kaur, G., Fekete-Szegӧ Inequality For Subclass Of Analytic Function Based On Generalized Derivative, *Aryabhatta Journal Of Mathematics And Informatics,* **9**(2017) *,*
38. Singh ,G. and Kaur, G., Coefficient Inequality For A Subclass Of Analytic Function using Subordination Method With Extremal Function, *International Journal Of Advance Research In Science And Engineering* , **7** (2018) , .
39. Singh ,G. and Garg, J., Coefficient Inequality For A New Subclass Of Analytic Functions, *Mathematical Sciences* *International Research Journal,* **4**(2015) *,*
40. Singh ,G. and Kaur, N., Fekete-Szegӧ Inequality For Certain Subclasses Of Analytic Functions, *Mathematical Sciences* *International Research Journal,* **4**(2015)
41. Singh. G, Singh. B, Fekete Szego Coefficient Inequality of Regular Functions for A Special Class, International Journal of Research in Engineering and Science (IJRES), 10 (8), 2022, 556-560
42. Singh. G, Rani M, An advance subclass of Analytic Functions having a unique coefficient inequality, International Journal of Research in Engineering and Science (IJRES), 10 (8), 2022, 474-476