##### FEKETE SZEGO INEQUALITY FOR A CLASS OF ANALYTIC FUNCTIONS APPROACHING TO CLASS OF CONVEX FUNCTIONS IN THE LIMIT FORM AND CLASS OF STARLIKE FUNCTIONS DIRECTLY

By **Gurmeet Singh**

Khalsa College Patiala-147001, Punjab, India

Email:meetgur111@gmail.com

**Abstract**

We introduce a class of analytic functions and obtain sharp upper bounds of the functional for the analytic function belonging to this class with special character that it tends to the class of convex functions as .

**Keywords:**Univalent functions, Starlike functions, Close to convex functions and bounded functions

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## 1 Introduction

Let denote the class of functions of the form

(1.1)

which are analytic in the unit disc . Let be the class of functions of the form (1.1), which are analytic univalent in .

In 1916, Bieber Bach [1, 2] proved that for the functions . In 1923, Löwner [10] proved that for the functions .

With the known estimates and , it was expected to try to find some relation between and for the class **,** Fekete and Szegö [4] used Löwner’s method to prove the following well known result for the class .

Let , then

(1.2)

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes [3, 9].

Let us define some subclasses of .

We denote by , the class of univalent starlike functions

and satisfying the condition

(1.3)

We denote by , the class of univalent convex functions

and satisfying the condition

(1.4)

A function is said to be close to convex if there exists such that

(1.5)

The class of close to convex functions is denoted by C and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

(1.6)

(1.7)

It is obvious that is a subclass of and is a subclass of .

We introduce a new subclass as

and we shall denote this class as

We shall deal with two subclasses of defined as follows in our next paper:

(1.8)

(1.9)

Symbol stands for subordination, which we define as follows:

**Principle of Subordination*.*** *Let and be two functions analytic in . Then is called subordinate to F(z) in if there exists a function analytic in satisfying the conditions and such that and we write*

By , we denote the class of analytic bounded functions of the form

(1.10)

It is known that

(1.11)

## 2 Preliminary Lemmas.

For , we write so that

(2.1)

## 3 Main Results

**Theorem 3.1**. Let

*The results are sharp.*

*Proof***.** By definition of , we have

(3.4)

Expanding the series (3.1), we get

(3.5)

Identifying terms in 3.2, we get

(3.6)

(3.7)

From (3.3) and (3.4), we obtain

(3.8)

Taking absolute value and using Triangular inequality, (3.5) can be rewritten as

(3.9)

Using (1.9) in (3.6), simple calculations yield

(3.10)

***Case* I.** . In this case, (3.10) can be rewritten as

(3.11)

***Subcase* I (a).**

Using (1.9), (3.8) becomes

(3.12)

***Subcase* I** **(b).**

We obtain from (3.8)

(3.13)

***Case* II*.***

Preceding as in case I, we get

(3.14)

***Subcase* II** **(a).**

(3.11) takes the form

(3.15)

Combining subcase I (b) and subcase II (a), we obtain

(3.16)

***Subcase* II (b).**

Preceding as in subcase I (a), we get

(3.17)

Combining (3.9), (3.13) and (3.14), the theorem is proved.

Extremal function for (3.1) and (3.3) is defined by

where

and

Extremal function for (3.2) is defined by ,

where and .

**Corollary 3.2.** *Putting and applying limit as in the theorem, we get*

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent convex functions.

**Corollary 3.3.** *Putting in the theorem, we get*

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

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