**ON THE INTEGRAL SOLUTIONS OF THE BINARY QUADRATIC EQUATION**

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**ABSTRACT**

**This paper concerns with the problem of obtaining infinitely many non-zero distinct integer solutions of the binary quadratic Diophantine equation . Employing the lemma of Brahmagupta, we find infinitely many integral solutions of the above equation. The recurrence relations on the solutions are presented. An interesting relations among the solutions are presented.**

**Key Words: Binary quadratic, Pell equation, integral solutions.**

**2010 mathematics subject classification: 11D09**

1. **INTRODUCTION**

 **Equations with integer co-efficient which are to be solved integer are called Diophanine equations. Consider the linear Diophantine equation has as a solution. In fact it has finitely many solutions , where t is an arbitrary integer. A Pell equation is a type of non-liear Diophantine equation in the form , where D is non-square positive integer. when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-10]. The above equation is also called the Pell Fermat equation.**

 **In this communication, we obtain infinitely many integer solutions to the Diophantine equation . Different relations among the solutions, different choices of hyperbolas, parabolas are also obtained.**

1. **METHODOLOGY**

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

 **** (1)

Introduce the linear transformations

  (2)

From (1) & (2) we have,

  (3)

The smallest positive integer solution is

 

Employing the integral solutions of the Pell equation

  (4)

And applying the lemma of Brahmagupta, the non-zero distinct integral solutions of (1) are found to be

  (5)

  (6)

Where

, 

The recurrence relations satisfied by  are correspondingly exhibited below.

 

, 

A few numerical examples are given in the following Table

**Table.1: Numerical examples**

|  |  |  |
| --- | --- | --- |
|  |  |  |
| -1 | 17 | 14 |
| 0 | 169 | 138 |
| 1 | 1673 | 1366 |
| 2 | 16561 | 13522 |

From the above table, we observe some interesting relations among the solutions which are presented below:

 (i) .  is always odd .

 (ii). is always even.

2.1 . A few interesting properties satisfied by the solutions of (1) are exhibited below:

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**2.2. Each of the following expressions represents a Nasty number:**

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**2.3. Each of the following expressions is a cubical integer:**

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**2.4. Each of the following expressions is a Bi-Quadratic integer:**

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**2.5. Each of the following expression is a Quintic integer:**

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**3.REMARKABLE OBSERVATIONS:**

* 1. Employing linear combinations among the solutions of (1.1), one may generateinteger solutions for other choices of hyperbola which are presented below:

**Table 2: Hyperbola**

|  |  |  |
| --- | --- | --- |
| **S.NO** | **Hyperbola** | **(X,Y)** |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |

* 1. Employing linear combinations among the solutions of (1.1), one may generateinteger solutions for other choices of parabola which are presented below:

**Table 3: Parabola**

|  |  |  |
| --- | --- | --- |
| **S.NO** | **Parabola** | **(X,Y)** |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |

* 1. Employing linear combinations among the solutions of (1.1), one may generateinteger solutions for other choices of straight line which are presented below:

**Table 4: Straight line**

|  |  |  |
| --- | --- | --- |
| **S.NO** | **Straight line** |  **(X,Y)** |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |

 **CONCLUSION**

In this paper, we have presented infinitely many integer solutions for the diophantine Equation**** . As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of the Diophantine Equations and determine their integer solutions along with suitable properties

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**Figure 1:** Pycnometer Test Procedure.

1. **RESULTS AND DISCUSSION**

In this Section results and discussion of the study is written. They may also be broken into subsets with short, revealing captions. This section should be typed in character size 10pt Times New Roman.

**Table 1.** Sample Comparison

|  |  |  |
| --- | --- | --- |
| SN. | Sample | Quantity (Liter) |
| 1 | Fluid A | 22 |
| 2 | Fluid-B | 15 |
| 3 | Fluid-C | 12 |
| 4 | Fluid-D | 10 |
| 5 | Fluid-E | 27 |
| 6 | Fluid-F | 32 |



**Figure 2:** 10 liter capacity vessel (Font size-10)

Unless or otherwise specified specific gravity values reported shall be based on water at 270C. So the specific gravity at 270C = K Sp. gravity at Tx0C. The specific gravity of the soil particles lie with in the range of 2.65 to 2.85. Soils containing organic matter and porous particles may have specific gravity values below 2.0. Soils having heavy substances may have values above 3.0.

1. **CONCLUSION**

All the main points of the research work are written in this section. Ensure that abstract and conclusion should not same. Graph and tables should not use in conclusion.

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