INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 05, Issue 03, March 2025, pp : 1683-1690
www.ijprems.com
editor@ijprems.com
e-ISSN : 2583-1062
Impact Factor : 7.001

APPLICATION OF QUEUING THEORY TO PASSENGERS' DEPARTURE

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DOI: https://www.doi.org/10.58257/IJPREMS39179

ABSRACT

Waiting in line is a very volatile situation in life which causes unnecessary delays and reduces the service effectiveness of an establishment. Queuing is a common phenomenon which occurs whenever the current demand of a service exceed the current capacity to provide for the service. This imbalance may only be temporary but during that temporary imbalanced queuing is formed. this study focus on determine the randomness of the arrival and departure of the departing passengers (passenger traveling out) at Murtala Muhammed International Airport (MMIA), Lagos Nigeria and a case study of Qatar Airways was used and the study also focus on mean arrival rate, average time a passenger spends in the system, number of passenger in the system and on the queue, Idle time of the airport attendant and traffic intensity, and study use observational method in collection of data recorded between the hours of 08:00 am and 12:00 pm at the Muritala Muhammed Airport, Lagos and it was concluded that that there exist no or less queue at the arrival counter on Qatar airway in Muritala Muhammed Airport, Lagos**.** Passengers are attended to almost immediately on their arrival. The maximum time a passenger spends on the system is few. The servers are efficient enough. For a more efficient service level, the number of servers could be increased.

Keywords: Queuing theory, waiting time, Poisson arrival, exponential service times

1. INTRODUCTION

Queuing theory is the mathematical theory of the formation and behavior of queues or waiting line being on a queue can sometimes be very frustrating and over the years queuing has been a great problem to the services rendering departments. Not only is it a problem to the institutions rendering these services, it is also a problem to the customers (passengers) to whom the service is rendered (since an average service seeker would not like the ideal of waiting on an endless queue before he/she is rendered the service he seeks). The airport being one of the services rendering out, it is not left out of this challenge. Some research as work on queue in hospital, bank, market and industries. Adeleke et al.(2009) worked on the application of queuing theory to waiting time of out-patients in hospitals The average number of patients, the average time spent by each patient as well as the probability of arrival of patients into the queuing system was obtained. Ogunwale and Olubiyi (2012) studied the comparative Analysis of Waiting Time of Customers in banks, Tsarouhas (2011) presented queuing theory to calculate the total processing time per pizza line at work station in food production line, Kumar and Jain (2013) studied threshold F- policy and N- policy for controlling the arrivals and service in the queuing Bakari et al. in (2014). Used supermarket, queuing process and its application to customer service delivery was proposed Azmat (2007) revealed the use of queuing theory in the analysis of the sales check out operation in a supermarket, Houda and Taoufik (2011) in there study used simulation of queuing theory in the toll motorway, Animatu et al. (2018) studied how the use of queuing theory in vehicular traffic flow can help in minimizing the delay on roads in the Kumasi, In 2014, Prasanta Kumar Brahma worked on using queuing theory and simulation model to optimize hospital central laboratory sample collection room. Afrane and Appah (2014) worked on queuing theory and the management of waiting time in hospitals Anglo Gold Asha.nti Hospital in Ghana. The study investigated the application of queuing theory and modelling to the queuing problems at the out- patient department of the hospital., Ademoh and Nneka (2014). worked on BDR modeling of passenger queues at Nnamdi Azikiwe International Airport, Abuja, Nigeria. The study developed a queuing model using the birth and death rate approach to simulate model for validation. Balakrishnan and Simaiavis (2015) worked on a queuing model of the airport departure process. This will focused on determine the randomness of the arrival and departure of the departing passengers.

2. METHODS

Queuing Equation For A Single Channel, Poisson Arrival And Exponential Service Times (M/M/1)

FOR EVENT A

A	Events	Probability
1	At time t, if (n-1) units are in the queue	$p_{n-1}(t)$
2	During the time Δt, only one unit arrives	$\lambda(\Delta t)$
3	During the time Δt, if no units is serviced	$1-\mu(\Delta t)$

Multiplying the three probabilities,

$p_{n-1}(t) \times \lambda(\Delta t) \times [1-\mu(\Delta t)]$

$p_n(t+\Delta t) = p_{n-1}(t)[\lambda(\Delta t) - \lambda\mu(\Delta t)^2]$

Since $(\Delta t)^2 \rightarrow 0$ as $t \rightarrow \infty$ we have;

$$p_n(t+\Delta t) = p_{n-1}(t)[\lambda(\Delta t) - \lambda\mu(0) = p_{n-1}(t)\lambda(\Delta t) \quad (1)$$

FOR EVENT B

B	Events	Probability
1	At time t, if (n+1) units are in the queue	$p_{n+1}(t)$
2	During the time Δt, if one is serviced	$\mu(\Delta t)$
3	During the time Δt, if no units arrives	$1-\mu(\Delta t)$

Multiplying the three probabilities,

$p_{n+1}(t) \times \mu(\Delta t) \times [1-\mu(\Delta t)]$

$p_n(t+\Delta t) = p_{n+1}(t)[\mu(\Delta t) - \lambda\mu(\Delta t)^2]$

Since $(\Delta t)^2 \rightarrow 0$ as $t \rightarrow \infty$ we have; $p_n(t+\Delta t) = p_{n+1}(t)\mu(\Delta t)$ (2)

FOR EVENT C

C	Events	Probability
1	At the t, if there are n units in the queue	$p_n(t)$
2	During the time Δt, one unit arrives	$\lambda(\Delta t)$
3	During the time Δt, one unit is serviced	$\mu(\Delta t)$

Multiplying the three probabilities,

$p_n(t) \times \lambda(\Delta t) \times \mu(\Delta t)$

Since $(\Delta t)^2 \rightarrow 0$ as $t \rightarrow \infty$ we have; $p_n(t+\Delta t) = p_n(t)\lambda\mu(\Delta t)^2 = 0$ (3)

FOR EVENT D

D	Events	Probability
1	T time t, if there are n units in the queue	$p_n(t)$
2	During the time Δt,	$1-\lambda(\Delta t)$
3	During the time Δt,	$1-\mu(\Delta t)$

Multiplying the three probabilities,

$p_n(t) \times [1-\lambda(\Delta t)] \times [1-\mu(\Delta t)]$

$p_n(t)[1-\mu(\Delta t) - \lambda(\Delta t) + \mu\lambda(\Delta t)^2]$

Since $(\Delta t)^2 \rightarrow 0$ as $t \rightarrow \infty$ we have;

$p_n(t+\Delta t) = p_n(t)[1-(\mu+\lambda)(\Delta t)]$ $p_n(t+\Delta t) = p_n(t) - p_n(t)(\mu+\lambda)\Delta t$ (4)

Adding the total probabilities for A, B, C, D.

$p_n(t+\Delta t) = p_{n-1}(t)\lambda(\Delta t) + p_{n+1}(t)\mu(\Delta t) + 0 + p_n(t) - p_n(t)(\mu+\lambda)\Delta t$

$p_n(t+\Delta t)+p_n(t)=p_{n-1}(t)\lambda(\Delta t)+p_{n+1}(t)\mu(\Delta t)+p_n(t)-p_n(t)(\mu+\lambda)\Delta t$

Dividing by(Δt),

$$\frac{p_n(t+\Delta t)+p_n(t)}{\Delta t}=\lambda p_{n-1}(t)+\mu p_{n+1}(t)-p_n(t)(\mu+\lambda)$$

Taking the limit as $(\Delta t)\to 0$

$$\delta\frac{p_n(t)}{\delta t}=\lambda p_{n-1}(t)+\mu p_{n+1}(t)-p_n(t)(\mu+\lambda)$$

At n=0

$$\delta\frac{p_o(t)}{\delta t}=\mu p_1(t)-p_o(t)-p_0(t)(\mu+\lambda)=\mu p_1(t)-\mu p_o(t)-\lambda p_o(t)$$

$$\delta\frac{p_o(t)}{\delta t}=\mu p_1(t)-\lambda p_0(t) \quad (5)$$

Setting the differential equation to zero, equation (3.6) is obtained as;

$\mu p_1-\lambda p_0=0$

$\mu p_1=\lambda p_0$ So that,

$$p_1=\frac{\lambda}{\mu}p_o=\rho p_0 \quad (6)$$

We can find that,

1. server utilization factor,

$$\rho=\frac{\lambda}{\mu} \quad (7)$$

2. average number of passengers in the system,

$$l_s=\frac{\rho}{1-\rho}=\frac{\lambda}{\mu-\lambda} \quad (8)$$

3. average number of queue,

$$l_q=l_s-\rho=\frac{\rho^2}{1-\rho} \quad (9)$$

4. average time in system,

$$w_s=\frac{1}{\mu-\lambda} \quad (10)$$

5. average time in queue,

$$w_q=\frac{\lambda}{\mu(\mu-\lambda)}=\frac{\rho}{(\mu-\lambda)} \quad (11)$$

6. probability of having o passengers in the system (i.e. the service unit is idle)

$$p_0=1-\frac{\lambda}{\mu}=i-\rho \quad (12)$$

7. probability of having n passengers in the system,

$$p_n=\rho^n(1-\rho) \quad (13)$$

From first principle,$y+\delta y-y$

The data displayed in the appendix was obtained by observational method.

From the appendix, it can be clearly noted that the time of arrival of each passenger, the time into service and the time out of service were recorded.

Analysis of the data was done using Queuing and Operation Research model toolpaks in Microsoft Excel Software package.

Table 2.1: Frequency Of Passenger's Arrival Within One Hour

	8.00am-9.00am	9.01am – 10.00am	10.01am- 11.00am	**TOTAL**
Day 1	40	40	20	100
Day 2	41	55	4	100
Day 3	43	50	7	100

Day 4	43	54	3	100
Day 5	40	60	-	100
Day 6	43	55	2	100
Day 7	50	50	-	100
Day 8	47	53	-	100
Day 9	46	54	-	100
Day 10	45	55	-	100
TOTAL	438	526	36	1000

From the data above, it can be deduced that passengers arrive mostly between 9.00 am to 10.00 am in the morning.

Table 2.2: Frequency Of Passenger's Arrival To Each Counter

	Counter A	Counter B	Counter C	Counter D	Counter E	Counter F
Day 1	19	16	20	16	17	12
Day 2	23	15	16	15	16	15
Day 3	18	17	13	23	16	13
Day 4	16	18	19	19	16	12
Day 5	16	20	17	21	13	13
Day 6	18	15	18	20	14	15
Day 7	19	21	19	17	11	13
Day 8	17	21	18	16	16	12
Day 9	20	18	18	17	14	13
Day 10	19	19	20	18	11	13

From the data above, it can be deduced that passengers arrive mostly to be checked on counters A to D (which are economic traveling passengers) while few passengers (which are VIPs) stay on counters E and F.

Table 2.3: Arrival Duration and Service Duration For The 10days

	Arrival time (minutes)	Service time (minutes)
Day 1	144	194
Day 2	121	187
Day 3	119	200
Day 4	123	197
Day 5	116	194
Day 6	111	186
Day 7	119	197
Day 8	110	188
Day 9	106	191
Day 10	110	184

Table 2.4: Arrival Rate And Service Rate For The 10 days

	Arrival Rate (per minute)	Service Rate (per minute)
Day 1	0.6944	0.5155
Day 2	0.8264	0.5348
Day 3	0.8403	0.5000

Day 4	0.8130	0.5076
Day 5	0.8621	0.5155
Day 6	0.9009	0.5376
Day 7	0.8403	0.5076
Day 8	0.9091	0.5319
Day 9	0.9434	0.5236
Day 10	0.9091	0.5435

Table 2.5: Mean Arrival Rate And Mean Service Rate

Mean Arrival Rate (λ)	0.853912 per minute
Mean Service Rate (μ)	0.52175 per minute
Number of Channels	6

Table 4.6: Queue Analysis

Queue Station	Que_4
Arrival Rate	0.853912
Service Rate/Channel	0.52175
Number of Servers	6
Max. Number in System	***
Number in Population	***
Type	M/M/6
Mean Number at Station	1.639309
Mean Time at Station	1.919763
Mean Number in Queue	0.002678
Mean Time in Queue	0.003137
Mean Number in Service	1.636631
Mean Time in Service	1.916627
Throughput Rate	0.853912
Efficiency	0.272772
Probability All Servers Idle	0.194551
Prob. All Servers Busy	0.007141
Prob. System Full	0
Critical Wait Time	1
P(Wait >= Critical Wait)	0.000733
P(0)	0.194551
P(1)	0.318408
P(2)	0.260558
P(3)	0.142146
P(4)	0.05816
P(5)	0.019037
P(6)	0.005193

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P(7)	0.001416
P(8)	0.000386
P(9)	0.000105
P(10)	2.87E-05

From Table 4.6 above, the average number of passengers in the system, L_s is 1.6393 ~ 2 passenger in the system at a time.

The average number of time a passenger spends in the system, W_s is 1.919 ~ 2 minutes.

The average number of passengers in the queue, L_q is 0.002678.

The average number of time a passenger spends in the queue waiting for service, W_q is 1.916627 minutes ~ 2 minutes

The utilization factor, ρ, is 0.2727 i.e. each server is busy for 27% of the time

The probability that all servers are idle, P_0 , i.e zero passengers on queue is 0.194551

The probability that all servers are busy is 0.007141

Table 4.5 above reveals that the probability that there are zero passenger on the queue is 0.194551 ~ 19%

The probability that there is one passenger on the queue is 0.318408 ~ 32%

The probability that there are two passengers on the queue is 0.260558 ~ 26%

The probability that there are three passengers on the queue is 0.142146 ~ 14%

The probability that there are four passengers on the queue is 0.05816 ~ 0.5%

The probability that there are five passengers on the queue is 0.019037~ 0.19%

The probability that there are six passengers on the queue is 0.005193 ~ 0.052%

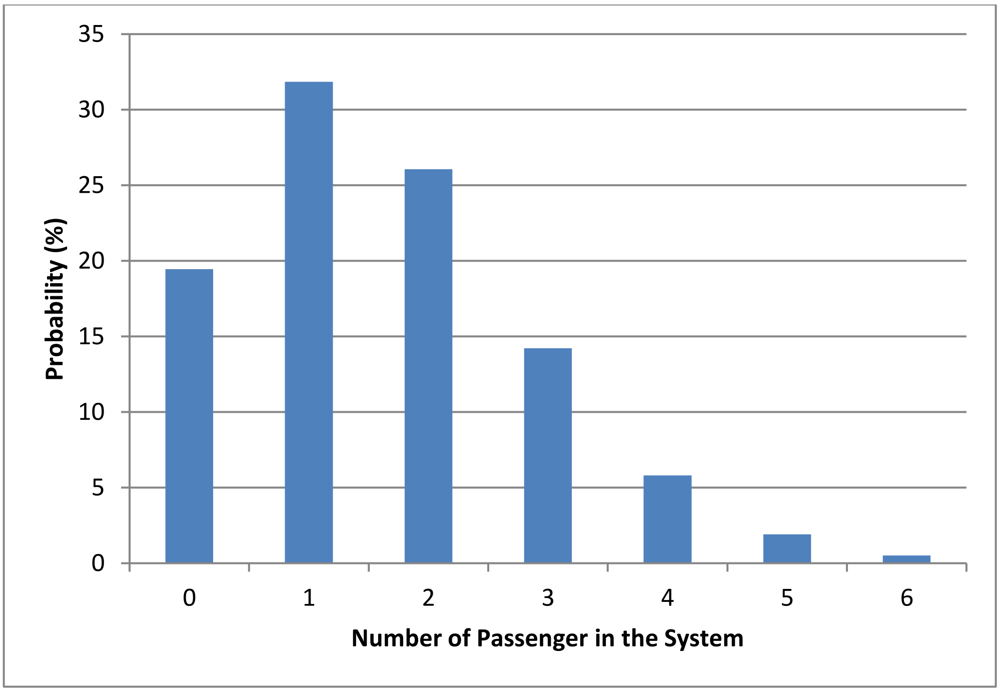

Figure 2.1: State probabilities

The percentage of passengers in this system that waits for a threshold time of 2 minutes or less before they are attended to:

QTPMMS_ServiceLevel (Threshold time, Arrival Rate, Service Rate, Servers)

QTPMMS_ServiceLevel(2,0.853912,0.52175,6)= 0.9999= 99%

If 99% of the waits 2 minutes or less before they are attended to, it means that 1% waits longer.

With the same threshold time of 2 minutes, we obtain the number of servers needed to achieve a service level of 99% .

QTPMMS_MinServers (Threshold_time, Serviceslevel, Arrival Rate, Service Rate)

QTPMMS_MinServers(2,99%,0.8539,0.5218) = 4

This implies that for 99% of the arriving passengers to spend a maximum of 2 minutes before being attended to, a minimum of 4 channels is sufficient.

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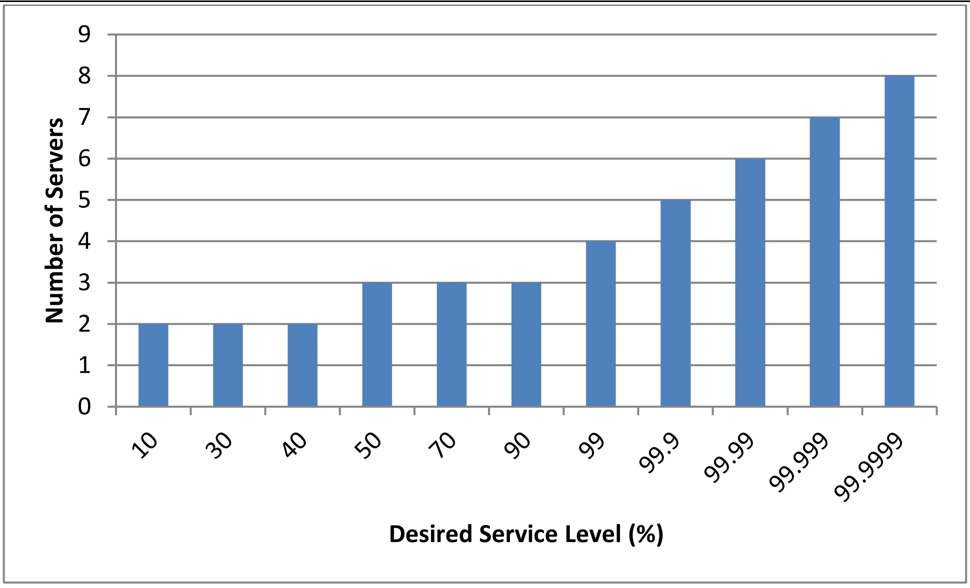

Fig 2.1: Graph showing the desired service levels with respect to the minimum number of the servers

This study focused on examining the arrival rate, waiting time and service rate of passengers. In this research work, attention is focused on the application of queuing theory to the passengers boarding Qatar airways at Muritala Muhammed Airport, Lagos.

The average number of passengers in the system at a time is two.

The average number of time a passenger spends in the system is two minutes.

The average number of passengers in the queue is 0.002678.

The average number of time a passenger spends in the queue waiting for service is 2 minutes.

The utilization factor, ρ is 27% i.e. each server is busy for 27% of the time.

The probability that all servers are idle, P_0, i.e. zero passengers on queue is 0.1946

The probability that all servers are busy is 0.0071

The probability that there are zero passenger on the queue is 19%

The probability that there is one passenger on the queue is 32%

The probability that there are two passengers on the queue is 26%

The probability that there are three passengers on the queue is 14%

The probability that there are four passengers on the queue is 0.5%

The probability that there are five passengers on the queue is 0.19%

The probability that there are six passengers on the queue is 0.052%

3. CONCLUSION

From the results obtained in this research work, it can be concluded that there exist no or less queue at the arrival counter on Qatar airway in Muritala Muhammed Airport, Lagos**.** Passengers are attended to almost immediately on their arrival. The maximum time a passenger spends on the system is few. The servers are efficient enough. For a more efficient service level, the number of servers could be increased.

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