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OPEN NEIGHBOURHOOD SOMBOR DEGREE BASED TOPOLOGICAL INDICES OF BASIC GRAPHS

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ABSTRACT

In this paper, we introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs such as Path, Cycle and Complete graphs.

Keywords: Open Neighbourhood, Degree based topological indices, Basic graphs.

1. INTRODUCTION

Let G be a simple, finite and connected graph with q vertices and r edges. The degree of a vertex in a graph G is denoted as $d(q)$. The first degree-based structure descriptors were conceived in the 1970s [3]. In 2019, S.Mondal et al.,[9,10] introduced the neighbourhood degree based topological indices. In 2021, V.Ravi et al.,[12] introduced some open neighbourhood degree based topological indices.

In 1975, Milan Randić introduced the Randić index [8]. In 1972, Gutman and Trinajsti´c introduced the first and second Zagreb indices [1,2]. In 2021, I.Gutman[5] introduced the Sombor index. In 2021, V.R.Kulli [7] introduced the Banhatti-Sombor index. In 2024, I.Gutman et al.,[4] discussed the Elliptic Sombor index. In 2021, I.Gutman introduced the Reduced Sombor index. In 2024, I.Gutman[5] discussed the Euler Sombor index. Motivated by the above studies, in this paper we introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs. Now, we discuss the Open Neighbourhood Sombor degree based topological indices of aforesaid, where the open neighbourhood index is given by $\alpha(q) = \sum_{q\in N_G(r)} d(q)$, $N_G(r)$ represents the neighbourhood of vertex r in the graph G and $d(q)$ denotes the degree of the vertex q.

- The **Open Neighbourhood Sombor Index** is defined as

$$N_oSO = \sum_{qr\in E(G)} \frac{\sqrt{\alpha(q)^2 + \alpha(r)^2}}{2}$$

- The **Open Neighbourhood Banhatti Sombor Index** is defined as

$$N_oBSO = \sum_{qr\in E(G)} \frac{\sqrt{\frac{1}{(\alpha(q))^2} + \frac{1}{(\alpha(r))^2}}}{2}$$

- The **Open Neighbourhood Elliptic Sombor Index** is defined as

$$N_oESO = \sum_{qr\in E(G)} \frac{(\alpha(q) + \alpha(r))\sqrt{\alpha(q)^2 + \alpha(r)^2}}{2}$$

- The **Open Neighbourhood Reduced Sombor Index** is defined as

$$N_oRSO = \sum_{qr\in E(G)} \frac{\sqrt{(\alpha(q) - 1)^2 + (\alpha(r) - 1)^2}}{2}$$

- The **Open Neighbourhood of Euler Sombor Index** is defined as

$$N_oEUSO = \sum_{qr\in E(G)} \frac{\sqrt{\alpha(q)^2 + \alpha(r)^2 + \alpha(q)\alpha(r)}}{2}$$

Also, we proposed reciprocal of Open Neighbourhoods Degree Sum Based Sombor Indices are given below

- The **Reciprocal Open Neighbourhood Sombor Index** is defined as

$$RN_oSO = \sum_{qr\in E(G)} \frac{2}{\sqrt{\alpha(q)^2 + \alpha(r)^2}}$$

- The **Reciprocal Open Neighbourhood Banhatti Sombor Index** is defined as

$$RN_oBSO = \sum_{qr\in E(G)} \frac{2}{\sqrt{\frac{1}{(\alpha(q))^2} + \frac{1}{(\alpha(r))^2}}}$$

- The **Reciprocal Open Neighbourhood Elliptic Sombor Index** is defined as

$$RN_oESO = \sum_{qr\in E(G)} \frac{2}{(\alpha(q) + \alpha(r))\sqrt{\alpha(q)^2 + \alpha(r)^2}}$$

- The **Reciprocal Open Neighbourhood Reduced Sombor Index** is defined as

$$RN_oRSO = \sum_{qr\in E(G)} \frac{2}{\sqrt{(\alpha(q) - 1)^2 + (\alpha(r) - 1)^2}}$$

- The **Reciprocal Open Neighbourhood of Euler Sombor Index** is defined as

$$RN_oESO = \sum_{qr\in E(G)} \frac{2}{\sqrt{\alpha(q)^2 + \alpha(r)^2 + \alpha(q)\alpha(r)}}$$

2. MAIN RESULTS

In this section we compute the Open Neighbourhood Sombor degree based topological indices of some basic graphs such as Path, Cycle and Complete graphs.

2.1 Open Neighbourhood Sombor Degree Based Topological Indices of Path graph

Let P_n be the Path graph on n vertices. The Open Neighbourhood Sombor index edge partitions for P_n are $(RRB(q), RRB(r)) = (2,3), (3,4), (4,4)$ Count= $2,2,(n-5)$.

Theorem:2.1.1

Let P_n be a path graph with $n \geq 5$ vertices. Then

a) $N_oSO(P_n) = \sqrt{13} + 5 + 2\sqrt{2}(n-5)$

b) $N_oBSO(P_n) = \frac{1}{2}\left(\frac{\sqrt{13}}{3} + \frac{5}{6} + \frac{(n-5)\sqrt{2}}{4}\right)$

c) $N_oESO(P_n) = 6\sqrt{13} + 35 + (n-5)8\sqrt{2}$

d) $N_oRSO(P_n) = \sqrt{5} + \sqrt{13} + (n-5)\frac{3\sqrt{2}}{2}$

e) $N_oEUSO(P_n) = \sqrt{19} + \sqrt{37} + (n-5)2\sqrt{3}$

f) $RN_oSO(P_n) = \frac{4}{\sqrt{13}} + \frac{4}{5} + \frac{(n-5)}{2\sqrt{2}}$

g) $RN_oBSO(P_n) = \frac{24}{\sqrt{13}} + \frac{48}{5} + \frac{8(n-5)}{\sqrt{2}}$

h) $RN_oESO(P_n) = \frac{4}{5\sqrt{13}} + \frac{4}{35} + \frac{(n-5)}{16\sqrt{2}}$

i) $RN_oRSO(P_n) = \frac{4}{\sqrt{5}} + \frac{4}{\sqrt{13}} + \frac{2(n-5)}{3\sqrt{2}}$

j) $RN_oEUSO(P_n) = \frac{4}{\sqrt{19}} + \frac{4}{\sqrt{37}} + \frac{(n-5)}{2\sqrt{3}}$

Proof:

(a) $N_oSO(P_n) = \sum_{qr\in E(G)} \frac{\sqrt{\alpha_{P_n}(q)^2 + \alpha_{P_n}(r)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j}\left(\frac{\sqrt{\alpha_{P_n}(q_i)^2 + \alpha_{P_n}(r_j)^2}}{2}\right)$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{\sqrt{2^2+3^2}}{2}\right) + \left(\frac{\sqrt{3^2+4^2}}{2}\right) + \left(\frac{\sqrt{4^2+4^2}}{2}\right)\right)$$

$$= 2\left(\frac{\sqrt{2^2+3^2}}{2}\right) + 2\left(\frac{\sqrt{3^2+4^2}}{2}\right) + (n-5)\left(\frac{\sqrt{4^2+4^2}}{2}\right)$$

$$= \sqrt{13} + 5 + 2\sqrt{2}(n-5)$$

(b) $N_oBSO(P_n) = \sum_{qr\in E(G)} \frac{\sqrt{\frac{1}{(\alpha_{P_n}(q))^2}+\frac{1}{(\alpha_{P_n}(r))^2}}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\frac{1}{(\alpha_{P_n}(q_i))^2}+\frac{1}{(\alpha_{P_n}(r_j))^2}}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{\sqrt{\frac{1}{(2)^2}+\frac{1}{(3)^2}}}{2}\right)+\left(\frac{\sqrt{\frac{1}{(3)^2}+\frac{1}{(4)^2}}}{2}\right)+\left(\frac{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}}{2}\right)\right)$$

$$= 2\left(\frac{\sqrt{\frac{1}{(2)^2}+\frac{1}{(3)^2}}}{2}\right) + 2\left(\frac{\sqrt{\frac{1}{(3)^2}+\frac{1}{(4)^2}}}{2}\right) + (n-5)\left(\frac{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{13}}{3}+\frac{5}{6}+\frac{(n-5)\sqrt{2}}{4}\right)$$

(c) $N_oESO(P_n) = \sum_{qr\in E(G)} \frac{(\alpha_{P_n}(q)+\alpha_{P_n}(r))\sqrt{\alpha_{P_n}(q)^2+\alpha_{P_n}(r)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{(\alpha_{P_n}(q_i)+\alpha_{P_n}(r_j))\sqrt{\alpha_{P_n}(q_i)^2+\alpha_{P_n}(r_j)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{(2+3)\sqrt{2^2+3^2}}{2}\right)+\left(\frac{(3+4)\sqrt{3^2+4^2}}{2}\right)+\left(\frac{(4+4)\sqrt{4^2+4^2}}{2}\right)\right)$$

$$= 2\left(\frac{(2+3)\sqrt{2^2+3^2}}{2}\right) + 2\left(\frac{(3+4)\sqrt{3^2+4^2}}{2}\right) + (n-5)\left(\frac{(4+4)\sqrt{4^2+4^2}}{2}\right)$$

$$= 6\sqrt{13} + 35 + (n-5)8\sqrt{2}$$

(d) $N_oRSO(P_n) = \sum_{qr\in E(G)} \frac{\sqrt{(\alpha_{P_n}(q)-1)^2+(\alpha_{P_n}(r)-1)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{(\alpha_{P_n}(q_i)-1)^2+(\alpha_{P_n}(r_j)-1)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{\sqrt{(2-1)^2+(3-1)^2}}{2}\right)+\left(\frac{\sqrt{(3-1)^2+(4-1)^2}}{2}\right)+\left(\frac{\sqrt{(4-1)^2+(4-1)^2}}{2}\right)\right)$$

$$= 2\left(\frac{\sqrt{(2-1)^2+(3-1)^2}}{2}\right) + 2\left(\frac{\sqrt{(3-1)^2+(4-1)^2}}{2}\right) + (n-5)\left(\frac{\sqrt{(4-1)^2+(4-1)^2}}{2}\right)$$

$$= \sqrt{5} + \sqrt{13} + (n-5)\frac{3\sqrt{2}}{2}$$

(e) $N_oEUSO(P_n) = \sum_{qr\in E(G)} \frac{\sqrt{\alpha_{P_n}(q)^2+\alpha_{P_n}(r)^2+\alpha_{P_n}(q)\alpha_{P_n}(r)}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\alpha_{P_n}(q_i)^2+\alpha_{P_n}(r_j)^2+\alpha_{P_n}(q_i)\alpha_{P_n}(r_j)}}{2}$$

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$$= \sum_{q_i}^{n} \sum_{r_j}^{n-5} \left(\left(\frac{\sqrt{2^2+3^2+2\times 3}}{2} \right) + \left(\frac{\sqrt{3^2+4^2+3\times 4}}{2} \right) + \left(\frac{\sqrt{4^2+4^2+4\times 4}}{2} \right) \right)$$

$$= 2\left(\frac{\sqrt{2^2+3^2+2\times 3}}{2} \right) + 2\left(\frac{\sqrt{3^2+4^2+3\times 4}}{2} \right) + (n-5)\left(\frac{\sqrt{4^2+4^2+4\times 4}}{2} \right)$$

$$= \sqrt{19} + \sqrt{37} + (n-5)2\sqrt{3}$$

(f) $RN_oSO(P_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{\alpha_{P_n}(q)^2+\alpha_{P_n}(r)^2}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{\sqrt{\alpha_{P_n}(q_i)^2 + \alpha_{P_n}(r_j)^2}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n-5} \left(\left(\frac{2}{\sqrt{2^2+3^2}} \right) + \left(\frac{2}{\sqrt{3^2+4^2}} \right) + \left(\frac{2}{\sqrt{4+4^2}} \right) \right)$$

$$= 2\left(\frac{2}{\sqrt{2^2+3^2}} \right) + 2\left(\frac{2}{\sqrt{3^2+4^2}} \right) + (n-5)\left(\frac{2}{\sqrt{4+4^2}} \right)$$

$$= \frac{4}{\sqrt{13}} + \frac{4}{5} + \frac{(n-5)}{2\sqrt{2}}$$

(g) $RN_oBSO(P_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{\frac{1}{(\alpha_{P_n}(q))^2}+\frac{1}{(\alpha_{P_n}(r))^2}}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{\sqrt{\frac{1}{(\alpha_{P_n}(q_i))^2} + \frac{1}{(\alpha_{P_n}(r_j))^2}}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n-5} \left(\left(\frac{2}{\sqrt{\frac{1}{(2)^2}+\frac{1}{(3)^2}}} \right) + \left(\frac{2}{\sqrt{\frac{1}{(3)^2}+\frac{1}{(4)^2}}} \right) + \left(\frac{2}{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}} \right) \right)$$

$$= 2\left(\frac{2}{\sqrt{\frac{1}{(2)^2}+\frac{1}{(3)^2}}} \right) + 2\left(\frac{2}{\sqrt{\frac{1}{(3)^2}+\frac{1}{(4)^2}}} \right) + (n-5)\left(\frac{2}{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}} \right)$$

$$= \frac{24}{\sqrt{13}} + \frac{48}{5} + \frac{8(n-5)}{\sqrt{2}}$$

(h) $RN_oESO(P_n) = \sum_{qr\in E(G)} \frac{2}{(\alpha_{P_n}(q)+\alpha_{P_n}(r))\sqrt{\alpha_{P_n}(q)^2+\alpha_{P_n}(r)^2}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{(\alpha_{P_n}(q_i) + \alpha_{P_n}(r_j))\sqrt{\alpha_{P_n}(q_i)^2 + \alpha_{P_n}(r_j)^2}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n-5} \left(\left(\frac{2}{(2+3)\sqrt{2^2+3^2}} \right) + \left(\frac{2}{(3+4)\sqrt{3^2+4^2}} \right) + \left(\frac{2}{(4+4)\sqrt{4^2+4^2}} \right) \right)$$

$$= 2\left(\frac{2}{(2+3)\sqrt{2^2+3^2}} \right) + 2\left(\frac{2}{(3+4)\sqrt{3^2+4^2}} \right) + (n-5)\left(\frac{2}{(4+4)\sqrt{4^2+4^2}} \right)$$

$$= \frac{4}{\sqrt{5}} + \frac{4}{\sqrt{13}} + \frac{2(n-5)}{3\sqrt{2}}$$

(i) $RN_oRSO(P_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{(\alpha_{P_n}(q)-1)^2+(\alpha_{P_n}(r)-1)^2}}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{2}{\sqrt{\left(\alpha_{P_n}(q_i)-1\right)^2+\left(\alpha_{P_n}(r_j)-1\right)^2}}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{2}{\sqrt{(2-1)^2+(3-1)^2}}\right)+\left(\frac{2}{\sqrt{(3-1)^2+(4-1)^2}}\right)+\left(\frac{2}{\sqrt{(4-1)^2+(4-1)^2}}\right)\right)$$

$$= 2\left(\frac{2}{\sqrt{(2-1)^2+(3-1)^2}}\right)+2\left(\frac{2}{\sqrt{(3-1)^2+(4-1)^2}}\right)+(n-5)\left(\frac{2}{\sqrt{(4-1)^2+(4-1)^2}}\right)$$

$$= \frac{4}{\sqrt{5}}+\frac{4}{\sqrt{13}}+\frac{2(n-5)}{3\sqrt{2}}$$

(j) $RN_oEUSO(P_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{\alpha_{P_n}(q)^2+\alpha_{P_n}(r)^2+\alpha_{P_n}(q)\alpha_{P_n}(r)}}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{2}{\sqrt{\alpha_{P_n}(q_i)^2+\alpha_{P_n}(r_j)^2+\alpha_{P_n}(q_i)\alpha_{P_n}(r_j)}}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n-5}\left(\left(\frac{2}{\sqrt{2^2+3^2+2\times 3}}\right)+\left(\frac{2}{\sqrt{3^2+4^2+3\times 4}}\right)+\left(\frac{2}{\sqrt{4^2+4^2+4\times 4}}\right)\right)$$

$$= 2\left(\frac{2}{\sqrt{2^2+3^2+2\times 3}}\right)+2\left(\frac{2}{\sqrt{3^2+4^2+3\times 4}}\right)+(n-5)\left(\frac{2}{\sqrt{4^2+4^2+4\times 4}}\right)$$

$$= \frac{4}{\sqrt{19}}+\frac{4}{\sqrt{37}}+\frac{(n-5)}{2\sqrt{3}}$$

2.2 Open Neighbourhood Sombor Degree Based Topological Indices of Cycle graph

Let C_n be the Cycle graph on n vertices. The Open Neighbourhood Sombor index edge partitions for C_n are $\left(RRB(q), RRB(r)\right) = (4,4)$ Count= n

Theorem:2.2.1

Let C_n be a Cycle graph with $n \geq 3$ vertices. Then

a) $N_oSO(C_n) = 2\sqrt{2}n$
b) $N_oBSO(C_n) = \frac{\sqrt{2}n}{8}$
c) $N_oESO(C_n) = 16\sqrt{2}n$
d) $N_oRSO(C_n) = \frac{3\sqrt{2}n}{2}$
e) $N_oEUSO(C_n) = 2n\sqrt{3}$
f) $RN_oSO(C_n) = \frac{n}{2\sqrt{2}}$
g) $RN_oBSO(C_n) = \frac{8n}{\sqrt{2}}$
h) $RN_oESO(C_n) = \frac{n}{16\sqrt{2}}$
i) $RN_oRSO(C_n) = \frac{2n}{3\sqrt{2}}$
j) $RN_oESO(C_n) = \frac{n}{2\sqrt{3}}$

Proof:

(a) $N_oSO(C_n) = \sum_{qr\in E(G)} \frac{\sqrt{\alpha_{C_n}(q)^2+\alpha_{C_n}(r)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j}\left(\frac{\sqrt{\alpha_{C_n}(q_i)^2+\alpha_{C_n}(r_j)^2}}{2}\right)$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{\sqrt{4^2+4^2}}{2}\right)\right)$$

$$= n\left(\frac{\sqrt{4^2+4^2}}{2}\right)$$
$$= 2\sqrt{2}n$$

(b) $N_oBSO(C_n) = \sum_{qr\in E(G)} \frac{\sqrt{\frac{1}{(\alpha_{C_n}(q))^2}+\frac{1}{(\alpha_{C_n}(r))^2}}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\frac{1}{(\alpha_{C_n}(q_i))^2}+\frac{1}{(\alpha_{C_n}(r_j))^2}}}{2}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}}{2}\right)\right)$$

$$= n\left(\frac{\sqrt{\frac{1}{(4)^2}+\frac{1}{(4)^2}}}{2}\right)$$

$$= \frac{\sqrt{2}n}{8}$$

(c) $N_oESO(C_n) = \sum_{qr\in E(G)} \frac{(\alpha_{C_n}(q)+\alpha_{C_n}(r))\sqrt{\alpha_{C_n}(q)^2+\alpha_{C_n}(r)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{(\alpha_{C_n}(q_i)+\alpha_{C_n}(r_j))\sqrt{\alpha_{C_n}(q_i)^2+\alpha_{C_n}(r_j)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{(4+4)\sqrt{4^2+4^2}}{2}\right)\right)$$

$$= n\left(\frac{(4+4)\sqrt{4^2+4^2}}{2}\right)$$

$$= 16\sqrt{2}n$$

(d) $N_oRSO(C_n) = \sum_{qr\in E(G)} \frac{\sqrt{(\alpha_{C_n}(q)-1)^2+(\alpha_{C_n}(r)-1)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{(\alpha_{C_n}(q_i)-1)^2+(\alpha_{C_n}(r_j)-1)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{\sqrt{(4-1)^2+(4-1)^2}}{2}\right)\right)$$

$$= n\left(\frac{\sqrt{(4-1)^2+(4-1)^2}}{2}\right)$$

$$= \frac{3\sqrt{2}n}{2}$$

(e) $N_oEUSO(C_n) = \sum_{qr\in E(G)} \frac{\sqrt{\alpha_{C_n}(q)^2+\alpha_{C_n}(r)^2+\alpha_{C_n}(q)\alpha_{C_n}(r)}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\alpha_{C_n}(q_i)^2+\alpha_{C_n}(r_j)^2+\alpha_{C_n}(q_i)\alpha_{C_n}(r_j)}}{2}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n} \left(\left(\frac{\sqrt{4^2 + 4^2 + 4 \times 4}}{2} \right) \right)$$

$$= n \left(\frac{\sqrt{4^2 + 4^2 + 4 \times 4}}{2} \right)$$

$$= 2n\sqrt{3}$$

(f) $RN_oSO(C_n) = \sum_{qr \in E(G)} \frac{2}{\sqrt{\alpha_{C_n}(q)^2 + \alpha_{C_n}(r)^2}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{\sqrt{\alpha_{C_n}(q_i)^2 + \alpha_{C_n}(r_j)^2}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n} \left(\left(\frac{2}{\sqrt{4^2 + 4^2}} \right) \right)$$

$$= n \left(\frac{2}{\sqrt{4^2 + 4^2}} \right)$$

$$= \frac{n}{2\sqrt{2}}$$

(g) $RN_oBSO(C_n) = \sum_{qr \in E(G)} \frac{2}{\sqrt{\frac{1}{(\alpha_{C_n}(q))^2} + \frac{1}{(\alpha_{C_n}(r))^2}}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{\sqrt{\frac{1}{(\alpha_{C_n}(q_i))^2} + \frac{1}{(\alpha_{C_n}(r_j))^2}}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n} \left(\left(\frac{2}{\sqrt{\frac{1}{(4)^2} + \frac{1}{(4)^2}}} \right) \right)$$

$$= n \left(\frac{2}{\sqrt{\frac{1}{(4)^2} + \frac{1}{(4)^2}}} \right)$$

$$= \frac{8n}{\sqrt{2}}$$

(h)$RN_oESO(C_n) = \sum_{qr \in E(G)} \frac{2}{(\alpha_{C_n}(q) + \alpha_{C_n}(r))\sqrt{\alpha_{C_n}(q)^2 + \alpha_{C_n}(r)^2}}$

$$= \sum_{q_i}^{n} \sum_{r_j} \frac{2}{(\alpha_{C_n}(q_i) + \alpha_{C_n}(r_j))\sqrt{\alpha_{C_n}(q)^2 + \alpha_{C_n}((r_j))^2}}$$

$$= \sum_{q_i}^{n} \sum_{r_j}^{n} \left(\left(\frac{2}{(4+4)\sqrt{4^2 + 4^2}} \right) \right)$$

$$= n \left(\frac{2}{(4+4)\sqrt{4^2 + 4^2}} \right)$$

$$= \frac{n}{16\sqrt{2}}$$

(i) $RN_oRSO(C_n) = \sum_{qr \in E(G)} \frac{2}{\sqrt{(\alpha_{C_n}(q) - 1)^2 + (\alpha_{C_n}(r) - 1)^2}}$

$$=\sum_{q_i}^{n}\sum_{r_j}\frac{2}{\sqrt{\left(\alpha_{C_n}(q_i)-1\right)^2+\left(\alpha_{C_n}(r_j)-1\right)^2}}$$

$$=\sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{2}{\sqrt{(4-1)^2+(4-1)^2}}\right)\right)$$

$$=n\left(\frac{2}{\sqrt{(4-1)^2+(4-1)^2}}\right)$$

$$=\frac{2n}{3\sqrt{2}}$$

(j) $RN_oEUSO(C_n)=\sum_{qr\in E(G)}\frac{2}{\sqrt{\alpha_{C_n}(q)^2+\alpha_{C_n}(r)^2+\alpha_{C_n}(q)\alpha_{C_n}(r)}}$

$$=\sum_{q_i}^{n}\sum_{r_j}\frac{2}{\sqrt{\alpha_{C_n}(q_i)^2+\alpha_{C_n}(r_j)^2+\alpha_{C_n}(q_i)\alpha_{C_n}(r_j)}}$$

$$=\sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{2}{\sqrt{4^2+4^2+4\times 4}}\right)\right)$$

$$=n\left(\frac{2}{\sqrt{4^2+4^2+4\times 4}}\right)$$

$$=\frac{n}{2\sqrt{3}}$$

2.3 Open Neighbourhood Sombor Degree Based Topological Indices of Complete graph

Let K_n be the Cycle graph on n vertices. The Open Neighbourhood Sombor index edge partitions for K_n are $\left(RRB(q),RRB(r)\right)=(2n-2,2n-2)$ Count= $\frac{n(n-1)}{2}$.

Theorem:2.3.1

Let K_n be a Complete graph with $n\geq 4$ vertices. Then

a) $N_oSO(K_n)=\sqrt{2}(n-1)^2$

b) $N_oBSO(K_n)=\left(\frac{n}{4\sqrt{2}}\right)$

c) $N_oESO(K_n)=2\sqrt{2}n(n-1)^2$

d) $N_oRSO(K_n)=\frac{\sqrt{2}n(2n^2-5n+3)}{4}$

e) $N_oEUSO(K_n)=n(n-1)^2$

f) $RN_oSO(K_n)=\frac{n}{2\sqrt{2}}$

g) $RN_oBSO(K_n)=\sqrt{2}n(n-1)^2$

h) $RN_oESO(K_n)=\frac{n}{8\sqrt{2}(n-1)}$

i) $RN_oRSO(K_n)=\frac{n(n-1)}{\sqrt{2}(2n-3)}$

j) $RN_oEUSO(K_n)=\frac{n}{2\sqrt{3}}$

Proof:

(a) $N_oSO(K_n)=\sum_{qr\in E(G)}\frac{\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2}}{2}$

$$=\sum_{q_i}^{n}\sum_{r_j}\left(\frac{\sqrt{\alpha_{K_n}(q_i)^2+\alpha_{K_n}(r_j)^2}}{2}\right)$$

$$=\sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{\sqrt{(2n-2)^2+(2n-2)^2}}{2}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{\sqrt{4^2+4^2}}{2}\right)$$

$$= \sqrt{2}(n-1)^2$$

(b) $N_oBSO(K_n) = \sum_{qr\in E(G)} \frac{\sqrt{\frac{1}{(\alpha_{K_n}(q))^2}+\frac{1}{(\alpha_{K_n}(r))^2}}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\frac{1}{(\alpha_{K_n}(q_i))^2}+\frac{1}{(\alpha_{K_n}(r_j))^2}}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{\sqrt{\frac{1}{(2n-2)^2}+\frac{1}{(2n-2)^2}}}{2}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{\sqrt{\frac{1}{(2n-2)^2}+\frac{1}{(2n-2)^2}}}{2}\right)$$

$$= \frac{n}{4\sqrt{2}}$$

(c) $N_oESO(K_n) = \sum_{qr\in E(G)} \frac{(\alpha_{K_n}(q)+\alpha_{K_n}(r))\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{(\alpha_{K_n}(q_i)+\alpha_{K_n}(r_j))\sqrt{\alpha_{K_n}(q_i)^2+\alpha_{K_n}(r_j)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{((2n-2)+(2n-2))\sqrt{(2n-2)^2+(2n-2)^2}}{2}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{((2n-2)+(2n-2))\sqrt{(2n-2)^2+(2n-2)^2}}{2}\right)$$

$$= 2\sqrt{2}n(n-1)^2$$

(d) $N_oRSO(K_n) = \sum_{qr\in E(G)} \frac{\sqrt{(\alpha_{K_n}(q)-1)^2+(\alpha_{K_n}(r)-1)^2}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{(\alpha_{K_n}(q_i)-1)^2+(\alpha_{K_n}(r_j)-1)^2}}{2}$$

$$= \sum_{q_i}^{n}\sum_{r_j}^{n}\left(\left(\frac{\sqrt{((2n-2)-1)^2+((2n-2)-1)^2}}{2}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{\sqrt{((2n-2)-1)^2+((2n-2)-1)^2}}{2}\right)$$

$$= \frac{\sqrt{2}n(2n^2-5n+3)}{4}$$

(e) $N_oEUSO(K_n) = \sum_{qr\in E(G)} \frac{\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2+\alpha_{K_n}(q)\alpha_{K_n}(r)}}{2}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{\sqrt{\alpha_{K_n}(q_i)^2+\alpha_{K_n}(r_j)^2+\alpha_{K_n}(q_i)\alpha_{K_n}(r_j)}}{2}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{\sqrt{(2n-2)^2+(2n-2)^2+(2n-2)\times(2n-2)}}{2}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{\sqrt{(2n-2)^2+(2n-2)^2+(2n-2)\times(2n-2)}}{2}\right)$$

$$= n(n-1)^2$$

(f) $RN_oSO(K_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2}}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{2}{\sqrt{\alpha_{K_n}(q_i)^2+\alpha_{K_n}(r_j)^2}}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{2}{\sqrt{(2n-2)^2+(2n-2)^2}}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{2}{\sqrt{(2n-2)^2+(2n-2)^2}}\right)$$

$$= \frac{n}{2\sqrt{2}}$$

(g) $RN_oBSO(K_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{\frac{1}{(\alpha_{K_n}(q)^2}+\frac{1}{(\alpha_{K_n}(r))^2}}}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{2}{\sqrt{\frac{1}{(\alpha_{K_n}(q_i))^2}+\frac{1}{(\alpha_{K_n}(r_j))^2}}}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{2}{\sqrt{\frac{1}{(2n-2)^2}+\frac{1}{(2n-2)^2}}}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{2}{\sqrt{\frac{1}{(2n-2)^2}+\frac{1}{(2n-2)^2}}}\right)$$

$$= \sqrt{2}n(n-1)^2$$

(h)$RN_oESO(K_n) = \sum_{qr\in E(G)} \frac{2}{(\alpha_{K_n}(q)+\alpha_{K_n}(r))\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2}}$

$$= \sum_{q_i}^{n}\sum_{r_j} \frac{2}{(\alpha_{K_n}(q_i)+\alpha_{K_n}(r_j))\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}((r_j))^2}}$$

$$= \sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{2}{((2n-2)+(2n-2))\sqrt{(2n-2)^2+(2n-2)^2}}\right)\right)$$

$$= \frac{n(n-1)}{2}\left(\frac{2}{((2n-2)+(2n-2))\sqrt{(2n-2)^2+(2n-2)^2}}\right)$$

$$= \frac{n}{8\sqrt{2}(n-1)}$$

(i) $RN_oRSO(K_n) = \sum_{qr\in E(G)} \frac{2}{\sqrt{(\alpha_{K_n}(q)-1)^2+(\alpha_{K_n}(r)-1)^2}}$

$$=\sum_{q_i}^{n}\sum_{r_j}\frac{2}{\sqrt{\left(\alpha_{K_n}(q_i)-1\right)^2+\left(\alpha_{K_n}(r_j)-1\right)^2}}$$

$$=\sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{2}{\sqrt{((2n-2)-1)^2+((2n-2)-1)^2}}\right)\right)$$

$$=\frac{n(n-1)}{2}\left(\frac{2}{\sqrt{((2n-2)-1)^2+((2n-2)-1)^2}}\right)$$

$$=\frac{n(n-1)}{\sqrt{2}(2n-3)}$$

(j) $RN_oEUSO(K_n)=\sum_{qr\in E(G)}\frac{2}{\sqrt{\alpha_{K_n}(q)^2+\alpha_{K_n}(r)^2+\alpha_{K_n}(q)\alpha_{K_n}(r)}}$

$$=\sum_{q_i}^{n}\sum_{r_j}\frac{2}{\sqrt{\alpha_{K_n}(q_i)^2+\alpha_{K_n}(r_j)^2+\alpha_{K_n}(q_i)\alpha_{K_n}(r_j)}}$$

$$=\sum_{q_i}^{n}\sum\nolimits_{r_j}^{n}\left(\left(\frac{2}{\sqrt{(2n-2)^2+(2n-2)^2+(2n-2)\times(2n-2)}}\right)\right)$$

$$=\frac{n(n-1)}{2}\left(\frac{2}{\sqrt{(2n-2)^2+(2n-2)^2+(2n-2)\times(2n-2)}}\right)$$

$$=\frac{n}{2\sqrt{3}}$$

3. CONCLUSION

In this paper, we have introduce and compute Open Neighbourhood Sombor degree based topological indices such as Open Neighbourhood Sombor Index , Open Neighbourhood Banhatti Sombor Index , Open Neighbourhood Elliptic Sombor Index, Open Neighbourhood Reduced Sombor Index , Open Neighbourhood of Euler Sombor Index, Reciprocal Open Neighbourhood Sombor Index, Reciprocal Open Neighbourhood Banhatti Sombor Index, Reciprocal Open Neighbourhood Elliptic Sombor Index, Reciprocal Open Neighbourhood Reduced Sombor Index, Reciprocal Open Neighbourhood of Euler Sombor Index for some standard graphs such as Path, Cycle and Complete graphs.

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