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PROPER D-LUCKY LABELING ON SOME EXTENDED DUPLICATE GRAPHS

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ABSTRACT

In this paper we investigate the existence of Proper D-Lucky labeling on Extended Duplicate graphs.

Keywords: Graph Labeling, D-Lucky Labeling, Path graph

1. INTRODUCTION

Esakkiammal et.al., [3] introduced the concept of **Proper D-lucky Labeling**. A d-lucky labeling is called proper if $l(u) \neq l(r)$ for every adjacent vertices u and r. The proper d-lucky number of a graph is the least positive integ k such that G has a Proper d-lucky labeling with $\{1,2,\dots,k\}$ as the set of labels and is denoted by $\eta_{pdl}(G)$.

Definition: 2.1.2 (Extended Duplicate Graph of Path)

Let $DG = (R_1, B_1)$ be the Duplicate graph of a Path $G(R,B)$. The Extended Duplicate Graph of Path EDG (P_m) is obtained from the duplicate graph of path by joining $r_2 r_2'$, .Clearly it has $2m+2$ vertices and $2m+1$ edges,[6].

Definition: 2.1.3 (Extended Duplicate Graph of Star)

Let $DG(S_m)$ be the duplicate graph of Star. The Extended Duplicate Graph of Star $EDG(S_m)$ is obtained from the duplicate graph of Star by joining $r_1 r_1'$. It has $6m$ vertices and $8m-3$ edges,[8].

Definition: 2.1.4 (Extended Duplicate Graph of Twig)

Let $DG(T_m)$ be the duplicate graph of Twig. The Extended Duplicate Graph of Twig $EDG(T_m)$ is obtained from the duplicate graph of Twig by joining $r_2 r_2'$. Cleary this EDG (T_m) has $6m+4$ vertices and $6m+3$ edges,[5].

Definition: 2.1.5 (Extended Duplicate Graph of Bistar)

Let DG $(BS_{m,m})$ be the duplicate graph of Bistar. The Extended Duplicate Graph of Bistar is obtained from the duplicate graph of Bistar by joining $r_1 r_1'$. Clearly EDG $(BS_{m,m})$ has $4m+4$ vertices and $4m+3$ edges,[7].

2. MAIN RESULT

In this section we investigate the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

THEOREM: 2.1

Extended Duplicate Graph of Path EDG (P_m) admits Proper d-lucky labeling with $\eta_{dl}(\text{EDG}\ (P_m)\ ) = 2$.

Proof: From the structure of EDG(P_m), it is clear that EDG(P_m) has $2m+2$ vertices and $2m+1$ edges. To prove EDG (P_m) is d-lucky, define the function $l: R(G) \to \mathbb{N}$ to label the vertices as follows:

For, $1 \leq i \leq m$

(i) $l\ (r_i) = 1$

(ii) $l\ (r_i') = 2$

From the structure of the EDG(P_m), it is clear that the degrees of the vertices are as follows:

(i) $d(r_1) = d(r_i') = d(r_{m+1}) = d(r_{m+1}') = 1$

(ii) $d(r_2) = d(r_2') = 3$

(iii) $d(r_i) = d(r_i') = 3, for\ 3 \leq i \leq m$

(i) $c(r_1) = c(r_{m+1}) = 3$

(ii) $c(r_1') = c(r_{m+1}') = 2$

(iii) $c(r_2) = 9$

(iv) $c(r_2') = 6$

(v) $c(r_i) = 9$, for $3 \leq i \leq m$

(vi) $c(r_i') = 4$, for $3 \leq i \leq m$

Clearly, c(u) ≠ c(v) for any two adjacent vertices of EDG(P_m).Therefore EDG(P_m) admits d-lucky labeling with $\eta_{dl}(\text{EDG}(P_m)) = 2$.

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EXAMPLE: 2.1

Proper D-lucky labeling EDG(P_5) is shown in the figure 2.1.1 respectively.

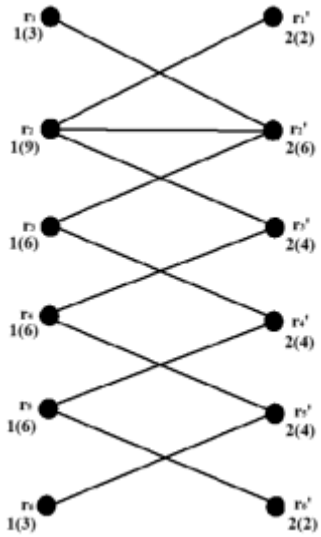

Figure 2.1.1

THEOREM 2.2

Extended duplicate Graph of Star EDG(S_m)admits Proper d-lucky labeling with $\eta_{dl}(\mathrm{ETG}(S_m)) = 2$.

Proof:

From the structure of EDG(S_m), it is clear that EDG(S_m) has 6m vertices and 8m-3 edges. To prove EDG(S_m) is d-lucky, define the function $l: R(G) \to \mathbb{N}$ to label the vertices as follows:

For, $1 \leq i \leq m$

(i) $l(r_i) = 1$

(ii) $l(r_i') = 2$

From the structure of the EDG(S_m), it is clear that the degrees of the vertices are as follows:

(iv) $d(r_1) = d(r_i') = m$,

(v) $d(r_i) = d(r_i') = 1, for\ 2 \leq i \leq m$

(vii) $c(r_1) = 3m$,

(viii) $c(r_1) = 3$, for $2 \leq i \leq m$

(ix) $c(r_1') = 2m$

(x) $c(r_1') = 2$, for $2 \leq i \leq m$

Clearly, $c(u) \neq c(r)$ for any two adjacent vertices of EDG(S_m). Therefore EDG(S_m) admits d-lucky labeling with $\eta_{dl}(\mathrm{EDG}\ (S_m)) = 2$.

EXAMPLE: 2.2

Proper D-lucky labeling EDG (S_5) is shown in the figure 2.2.1 respectively.

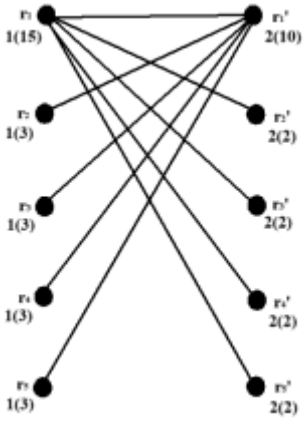

Figure 2.2.1

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THEOREM: 2.3

Extended Duplicate Graph of Twig EDG(T_m) admits Proper d-lucky labeling with $\eta_{dl}(\text{EDG}\ (T_m)\ ) = 2$.

Proof:

From the structure of EDG(T_m), it is clear that EDG(T_m) has $6m + 4$ vertices and $6m + 3$ edges. To prove EDG(T_m) is d-lucky, define the function $l: R(G) \to \mathbb{N}$ to label the vertices as follows:

For, $1 \le i \le 3m + 2$

(i) $l\ (r_i) = 1$

(ii) $l\ (r_i') = 2$

From the structure of the EDG(T_m), it is clear that the degrees of the vertices are as follows:

(i) $d(r_1) = 1, d(r_2) = 5$

(ii) $d(r_{3i}) = 1, for\ 1 \le i \le m$

(iii) $d(r_{3i+1}) = 1, for\ 1 \le i \le m$

(iv) $d(r_{3i+2}) = 4, for\ 1 \le i \le m - 1$

(v) $d(r_i') = 1, d(r_2') = 5$

(vi) $d(r_{3i}') = 1, for\ 1 \le i \le m$

(vii) $d(r_{3i+1}') = 1, for\ 1 \le i \le m$

(viii) $d(r_{3i+2}') = 4, for\ 1 \le i \le m - 1$

(ix) $d(r_{3m+2}) = d(r_{3m+2}') = 1$

(xi) $c(r_1) = 3, c(r_2) = 15$

(xii) $c(r_1') = 2, c(r_2') = 10$

(xiii) $c(r_{3i}) = c(r_{3i+1}) = 3, for\ 1 \le i \le m$

(xiv) $c(r_{3i}') = c(r_{3i+1}') = 2, for\ 1 \le i \le m$

(xv) $c(r_{3i+2}) = 12, c(r_{3i+2}') = 8, for\ 1 \le i \le m - 1$

Clearly, $c(u) \ne c(r)$ for any two adjacent vertices of EDG (T_m). Therefore EDG (T_m) admits d-lucky labeling with $\eta_{dl}(\text{EDG}\ (T_m)) = 2$.

EXAMPLE: 2.3

Proper D-lucky labeling EDG(T_3) is shown in the figure 2.3.1 respectively.

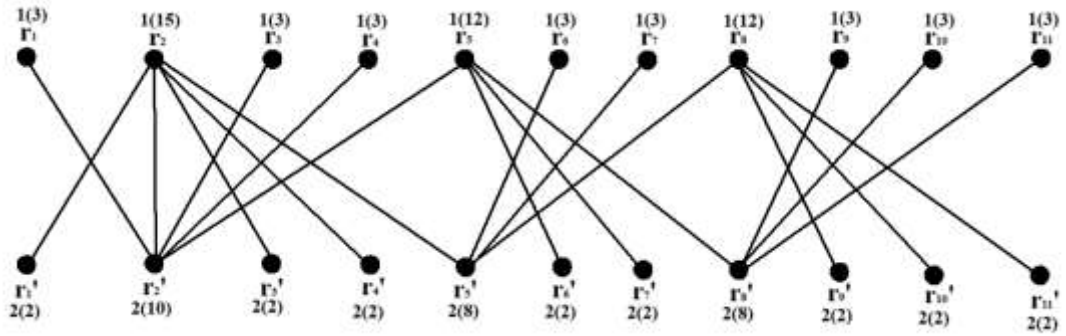

Figure 2.3.1

THEOREM: 2.4

Extended Duplicate Graph of Bistar EDG $(BS_{m,m})$ admits Proper d-lucky labeling with $\eta_{dl}(\text{EDG}\ (BS_{m,m})\ ) = 2$.

Proof:

From the structure of EDG $(BS_{m,m})$, it is clear that EDG $(BS_{m,m})$ has $4m + 4$ vertices and $4m + 3$ edges. To prove EDG $(BS_{m,m})$ is d-lucky, define the function $l: R(G) \to \mathbb{N}$ to label the vertices as follows:

For, $1 \le i \le 2m + 2$

(i) $l\ (r_i) = 1$

(ii) $l\ (r_i') = 2$

From the structure of the EDG (P_m), it is clear that the degrees of the vertices are as follows:

(i) $d(r_1) = d(r_i') = m + 2,$

(ii) $d(r_i) = d(r_i') = 1, for\ 2 \le i \le m + 1$

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(iii) $d(r_{m+2}) = d(r'_{m+2}) = m+1, for\ 1 \le i \le m$
(iv) $d(r_{m+2+i}) = d(r'_{m+2+i}) = 1, for\ 1 \le i \le m$
(v) $c(r_1) = 3m+6, c(r_i) = 3, for\ 2 \le i \le m+1$
(vi) $c(r_{m+2}) = 3m+3$
(vii) $c(r_{m+2+i}) = 3, for\ 1 \le i \le m$
(viii) $c(r'_1) = 2m+4, c(r'_{m+2}) = 2m+2$
(ix) $c(r'_i) = 2, for\ 2 \le i \le m+1$
(x) $c(r'_{m+2+i}) = 2, for\ 1 \le i \le m$

Clearly, $c(u) \neq c(r)$ for any two adjacent vertices of EDG $(BS_{m,m})$.Therefore EDG $(BS_{m,m})$ admits d-lucky labeling with $\eta_{dl}\left(\text{EDG}\left(BS_{m,m}\right)\right) = 2$.

EXAMPLE: 2.4

Proper D-lucky labeling EDG $(BS_{4,4})$ is shown in the figure 2.4.1 respectively.

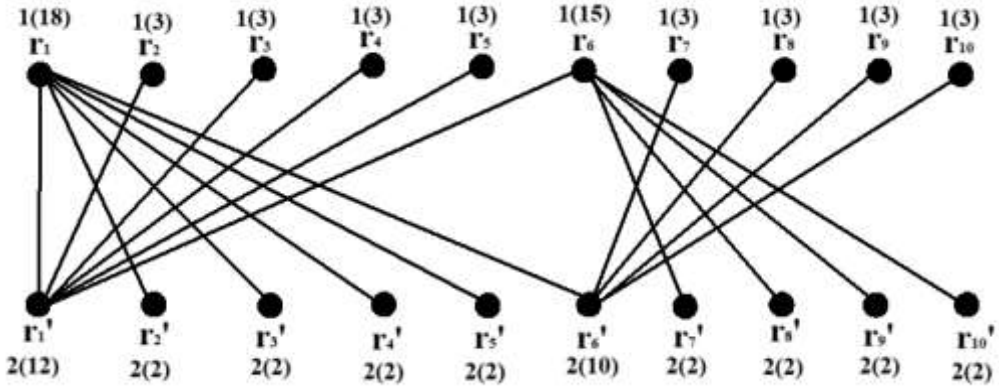

Figure 2.4.1

3. CONCLUSION

In this paper, we have confirmed the existence of Proper D-Lucky Labeling on Extended Duplicate Graph of Path, Extended Duplicate Graph of Star, Extended Duplicate Graph of Twig, Extended Duplicate Graph of Bistar graph.

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