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COEFFICIENT INEQUALITY MAKING RESULTS SHARP FOR CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS

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ABSTRACT

We have introduced subclasses of analytic functions and have obtained sharp upper bounds of the Fekete Szego functional $|a_3 - \mu a_2^2|$ for the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ belonging to these classes and subclasses.

Keywords: Univalent functions, Starlike functions, Close to convex functions and bounded functions.

1. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

analytic in the unit disc given by $\mathbb{E} = \{z: |z| < 1|\}$. Let $\mathcal{S}$ be the class of analytic functions of the form (1.1), which are univalent in $\mathbb{E}$. In 1916, Bieber Bach ([1], [2]) proved that $|a_2| \leq 2$ for the functions $f(z) \in \mathcal{S}$. In 1923, Löwner [10] proved that $|a_3| \leq 3$ for the functions $f(z) \in \mathcal{S}$..

With the known estimates $|a_2| \leq 2$and $|a_3| \leq 3$, it was natural to seek some relation between a_3 and ${a_2}^2$ for the class $\mathcal{S}$,Fekete and Szegö [4] used Löwner's method to prove the following well known result for the class $\mathcal{S}$.

Let $f(z) \in \mathcal{S}$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if\ \mu \leq 0; \\ 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right), if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if\mu \geq 1. \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes $\mathcal{S}$([3], [9]).

Let us define some subclasses of $\mathcal{S}$.

We denote by S*, the class of univalent starlike functions

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A} \text{ and satisfying the condition}$$

$$Re\left(\frac{zg(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.3}$$

We denote by $\mathcal{K}$, the class of univalent convex functions and satisfying the condition

$$h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in \mathcal{A}$$

$$Re\frac{((zh'(z))}{h'(z)} > 0, z \in \mathbb{E}. \tag{1.4}$$

A function $f(z) \in \mathcal{A}$ is said to be close to convex if there exists $g(z) \in S^*$ such that

$$Re\left(\frac{zf\prime(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.5}$$

The class of close to convex functions is denoted by C and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

$$S^*(A, B) = \left\{f(z) \in \mathcal{A}; \frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz}, -1 \leq B < A \leq 1, z \in \mathbb{E}\right\} \tag{1.6}$$

$$\mathcal{K}(A, B) = \left\{f(z) \in \mathcal{A}; \frac{(zf'(z))\prime}{f\prime(z)} \prec \frac{1+Az}{1+Bz}, -1 \leq B < A \leq 1, z \in \mathbb{E}\right\} \tag{1.7}$$

It is obvious that $S^*(A, B)$ is a subclass of S^* and $\mathcal{K}\ (A, B)$ is a subclass of $\mathcal{K}$.

Several authors studied and introduced various classes and subclasses of univalent analytic functions and established Fekete Szego inequality for the same. ([3]-[9], [12]-15], [22]-[62])

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N. Kaur [11] introduced a new subclass as and have established its coefficient inequality.

$$S^*(f,f',\alpha,\beta)=\left\{f(z)\in\mathcal{A};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta}\prec\frac{1+z}{1-z};z\in\mathbb{E}\right\}$$

We will deal with the subclass of $S^*(f,f',\alpha,\beta)$ defined as follows in the present paper:

$$S^*(f,f',\alpha,\beta,A,B)=\left\{f(z)\in\mathcal{A};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta}\prec\frac{1+Az}{1+Bz};z\in\mathbb{E}\right\}\quad(1.8)$$

We will deal with the subclass $S^*(f,f',\alpha,\beta,\delta)$ defined as follows in our next paper:

$$S^*(f,f',\alpha,\beta,\delta)=\left\{f(z)\in\mathcal{A};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta}\prec\left(\frac{1+z}{1-z}\right)^{\delta};z\in\mathbb{E}\right\}\quad(1.9)$$

Symbol $\prec$ stands for subordination, which we define as follows:

Principle of Subordination: Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to F(z) in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0)=0$ and $|w(z)|<1$ such that $f(z)=F(w(z));\ z\in\mathbb{E}$ and we write $f(z)\prec F(z)$.

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form $w(z)=\sum_{n=1}^{\infty}d_nz^n, w(0)=0, |w(z)|<1$. (1.10)

It is known that $|d_1|\le 1, |d_2|\le 1-|d_1|^2$. (1.11)

PRELIMINARY LEMMAS:

For $0<c<1$, we write $w(z)=\left(\frac{c+z}{1+cz}\right)$ so that

$$\frac{1+w(z)}{1-w(z)}=1+2cz+2z^2+\cdots.\quad(2.1)$$

2. MAIN RESULTS

THEORE 2.1

Let $f(z)\in S^*(f,f',\alpha,\beta,A,B)$, then The results are sharp.

$$|a_3-\mu a_2^2|$$

$$\le\begin{cases}\dfrac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}-\dfrac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu, \\ if\mu\le\dfrac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}; & (3.1)\\ \dfrac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)} \\ if\dfrac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}\le\mu\le \\ \dfrac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}; & (3.2)\\ \dfrac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu-\dfrac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}, \\ if\ \mu\ge\dfrac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} & (3.3\end{cases}$$

Proof 2.2. By definition of $S^*(f,f',\alpha,\beta,A,B)$, we have

$$(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta}=\frac{1+Aw(z)}{1+Bw(z)};w(z)\in\mathcal{U}.\quad(3.4)$$

Expanding the series (3.4), we get

$$(1-\alpha)\left\{1+\beta a_2z+(2\beta a_3+\frac{\beta(\beta-3)}{2}a_2^2)z^2+---\right\}+\alpha\{1+2(1-\beta)a_2z+2(1-\beta)(3a_3-(\beta+2)a_2^2)z^2+---\}=(1+(A-B)c_1z+(A-B)(c_2-Bc_1{}^2)z^2+---).\quad(3.5)$$

Identifying terms in (3.5), we get

$$a_2=\frac{(A-B)}{(1-\alpha)\beta+2\alpha(1-\beta)}c_1 \quad (3.6)$$

$$a_3=\frac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)}c_2+\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{c_1}^2. \quad (3.7)$$

From (3.6) and (3.7), we obtain

$$a_3-\mu a_2^2=\frac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)}c_2+\left[\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}-\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu\right]c_1^2. \quad (3.8)$$

Taking absolute value, (3.8) can be rewritten as

$$|a_3-\mu a_2^2|\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}|c_2|+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right||c_1^2|. \quad (3.9)$$

Using (1.9) in (3.9), we get

$$|a_3-\mu a_2^2|\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}(1-|c_1|^2)+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right||c_1^2$$

$$=\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right|-\frac{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(A-B)(3\alpha+\beta-4\alpha\beta)}\right]|c_1|^2. \quad (3.10)$$

Case I: $\mu\ \le\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$.

(3.10) can be rewritten as

$$|a_3-\mu a_2^2\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}-\mu\right]|c_1|^2.$$

(3.11)

Subcase I (a): $\mu\ \le\frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}$.

Using (1.9), (3.11) becomes

$$|a_3-\mu a_2^2|\le\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}-\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu \quad (3.12)$$

Subcase I (b): $\mu\ \ge\frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}$.

We obtain from (3.11)

$$|a_3-\mu a_2^2\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}. \quad (3.13)$$

Case II: $\mu\ \ge\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$

Preceding as in case I, we get

$$|a_3-\mu a_2^2\le\frac{1}{3\alpha+\beta-4\alpha\beta}+\frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\mu-\frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\right]|c_1|^2. \quad (3.14)$$

Subcase II (a): $\mu\ \le\frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}$

(3.14) takes the form

$$|a_3-\mu a_2^2\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta} \quad (3.15)$$

Combining subcase I (b) and subcase II (a), we obtain

$$|a_3-\mu a_2^2|\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}\,if\,\frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}\le\mu\le\frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} \quad (3.16)$$

Subcase II (b): $\mu\ \ge\frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}$

Preceding as in subcase I (a), we get

$$|a_3-\mu a_2^2|\le\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu-\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2} \quad (3.17)$$

Combining (3.12), (3.16) and (3.17), the theorem is proved.

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Corollary 2.3: Putting $\alpha = 1, \beta = 0, A = 1, B = -1$ in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 1-\mu, if \mu \leq 1; \\ \frac{1}{3} if 1 \leq \mu \leq \frac{4}{3}; \\ \mu - 1, if \ \mu \geq \frac{4}{3} \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent convex functions.

Corollary 2.3: Putting $\alpha = 0, \beta = 1, A = 1 , B = -1$ in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3-4\mu, if \mu \leq \frac{1}{2}; \\ 1 if \frac{1}{2} \leq \mu \leq 1; \\ 4\mu - 3, if \ \mu \geq 1 \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

Corollary 2.4: Putting $A = 1 , B = -1$ in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{\{(1-\alpha)\beta + 2\alpha(1-\beta)\}^2}\left[\frac{8\alpha + 3\beta + 4\alpha^2 - 12\alpha^2\beta - 9\alpha\beta^2 - 7\alpha\beta}{(3\alpha + \beta - 4\alpha\beta)} - 4\mu\right], \\ if \mu \leq \frac{8\alpha + 3\beta + 4\alpha^2 - \beta^2 - 3\alpha\beta^2 - 7\alpha\beta}{4(3\alpha + \beta - 4\alpha\beta)}; \\ \frac{1}{3\alpha + \beta - 4\alpha\beta} \\ if \frac{8\alpha + 3\beta + 4\alpha^2 - \beta^2 - 3\alpha\beta^2 - 7\alpha\beta}{4(3\alpha + \beta - 4\alpha\beta)} \leq \mu \leq \\ \frac{8\alpha + 3\beta + 8\alpha^2 + \beta^2 - 24\alpha^2\beta - 6\alpha\beta^2 - 7\alpha\beta}{4(3\alpha + \beta - 4\alpha\beta)}; \\ \frac{1}{\{(1-\alpha)\beta + 2\alpha(1-\beta)\}^2}\left[4\mu - \frac{8\alpha + 3\beta + 4\alpha^2 - 12\alpha^2\beta - 9\alpha\beta^2 - 7\alpha\beta}{(3\alpha + \beta - 4\alpha\beta)}\right], \\ if \ \mu \geq \frac{8\alpha + 3\beta + 8\alpha^2 + \beta^2 - 24\alpha^2\beta - 6\alpha\beta^2 - 7\alpha\beta}{4(3\alpha + \beta - 4\alpha\beta)} \end{cases}$$

These estimates were derived by N. Kaur [11] and are results for the subclass $S^*(f, f', \alpha, \beta)$ of univalent starlike functions.

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