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On Finding Integer Solutions To Non-Homogeneous Ternary Cubic Diophantine Equation $x^2+y^2-xy=z^3$

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ABSTRACT

Varieties Of Integer Solutions To Non-Homogeneous Ternary Cubic Equation Given By $x^2+y^2-xy=z^3$ Is Obtained Through Substitution Technique And Factorization Method.

Keywords: Non-Homogeneous Cubic , Ternary Cubic , Integer Solutions, Substitution Technique, Factorization Method

1. INTRODUCTION

The Theory Of Diophantine Equations Is An Ancient Subject That Typically Involves Solving, Polynomial Equation In Two Or More Variables Or A System Of Polynomial Equations With The Number Of Unknowns Greater Than The Number Of Equations, In Integers And Occupies A Pivotal Role In The Region Of Mathematics.The Subject Of Diophantine Equations Has Fascinated And Inspired Both Amateurs And Mathematicians Alike And So They Merit Special Recognition. Solving Higher Degree Diophantine Equations Can Be Challenging As They Involve Finding Integer Solutions That Satisfy The Given Polynomial Equation. Learning About The Various Techniques To Solve These Higher Power Diophantine Equation In Successfully Deriving Their Solutions Help Us Understand How Numbers Work And Their Significance In Different Areas Of Mathematics And Science. For The Sake Of Clear Understanding By The Readers, One May Refer The Varieties Of Cubic Diophantine Equations With Multi Variables [1-17]. This Paper Aims At Determining Many Integer Solutions To Non-Homogeneous Polynomial Equation Of Degree Three With Three Unknowns Given By $x^2-xy+y^2=z^3$. A Few Relations Between The Solutions Are Presented.

Methodology

The Non-Homogeneous Ternary Cubic Equation Under Consideration Is

$$x^2+y^2-xy=z^3 \tag{1}$$

Various Choices Of Integer Solutions To (1) Are Illustrated Below:

Choice 1

The Option

$$x=ky, k\geq 1 \tag{2}$$

In (1) Gives

$$(k^2-k+1)y^2=z^3$$

Which Is Satisfied By

$$y=(k^2-k+1)\alpha^{3s}, z=(k^2-k+1)\alpha^{2s}, \alpha>1, s\geq 0 \tag{3}$$

From (2) , We Get

$$x=k(k^2-k+1)\alpha^{3s} \tag{4}$$

Thus , (3) & (4) Satisfy (1).

Choice 2

The Option

$$x=u+kz, y=u-kz, u\neq kz \tag{5}$$

In (1) Leads To The Non-Homogeneous Cubic Equation

$$u^2 = z^2 (z - 3k^2) \qquad (6)$$

The R.H.S. Of (6) Is A Perfect Square When

$$z = (s^2 + 3)\, k^2 \qquad (7)$$

From (6) , We Have

$$u = s(s^2 + 3)k^3 \qquad (8)$$

Employing (7) & (8) In (5) ,It Is Seen That

$$\begin{aligned} x &= k^3 (s^2 + 3)(s + 1)\,, \\ y &= k^3 (s^2 + 3)(s - 1)\,, s > 1 \end{aligned} \qquad (9)$$

Thus , (7) & (9) Satisfy (1) .

Note 1

The R.H.S. Of (6) Is Also A Perfect Square For Values Of Z Given By

$$z_n = n^2 + 2kn + 4k^2 \qquad (10)$$

From (6) , We Get

$$u_n = (n^2 + 2kn + 4k^2)\,(n + k) \qquad (11)$$

In View Of (5) , We Have

$$\begin{aligned} x_n &= (n^2 + 2kn + 4k^2)\,(n + 2k)\,, \\ y_n &= (n^2 + 2kn + 4k^2)\,(n). \end{aligned} \qquad (12)$$

Thus , (10) & (12) Satisfy (1) .

Choice 3

The Transformation

$$x = kz + v, y = kz - v, v \neq kz \qquad (13)$$

In (1) Gives

$$v^2 = \frac{z^2 (z - k^2)}{3} \qquad (14)$$

The R.H.S. Of (14) Is A Perfect Square When

$$z = (3s^2 + 1)k^2 \qquad (15)$$

From (14) ,We Get

$$v = s\,(3s^2 + 1)\,k^3 \qquad (16)$$

Using (15) & (16) In (13) , We Have

$$\begin{aligned} x &= k^3 (3s^2 + 1)\,(1 + s)\,, \\ y &= k^3 (3s^2 + 1)\,(1 - s)\,, s \neq 1 \end{aligned} \qquad (17)$$

Thus , (15) & (17) Satisfy (1).

Choice 4

Introduction Of The Transformations

$$x = u + v, y = u - v, u \neq v \qquad (18)$$

In (1) Simplifies To The Non-Homogeneous Ternary Cubic Equation

$$u^2 + 3v^2 = z^3 \qquad (19)$$

Which Is Satisfied By

$$u = m(m^2 + 3n^2)\,, v = n(m^2 + 3n^2) \qquad (20)$$

And

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$$z = m^2 + 3n^2 \tag{21}$$

Substituting (20) In (18) , We Have

$$x = (m^2 + 3n^2)\,(m+n)\,,$$
$$y = (m^2 + 3n^2)\,(m-n)\,. \tag{22}$$

Thus , (21) & (22) Satisfy (1).

Note 2

It Is To Be Noted That (19) Is Also Satisfied By

$$u = m^3 - 9mn^2\,, v = 3m^2 n - 3n^3\,, z = m^2 + 3n^2$$

In This Case , The Integer Solutions To (1) Are Given By

$$x = m^3 - 9mn^2 + 3m^2 n - 3n^3\,,$$
$$y = m^3 - 9mn^2 - 3m^2 n + 3n^3\,,$$
$$z = m^2 + 3n^2\,.$$

Choice 5

The Option

$$x = u + v, y = u - v, z = v, u \neq v \tag{23}$$

In (1) Gives

$$u^2 = v^2\,(v-3) \tag{24}$$

The R.H.S. Of (24) Is A Perfect Square When

$$v = s^2 + 3 \tag{25}$$

Using (25) In (24) ,We Have

$$u = s\,(s^2 + 3)$$

In View Of (23) , The Integer Solutions To (1) Are Given By

$$x = (s^2 + 3)(s+1)\,,$$
$$y = (s^2 + 3)(s-1)\,,$$
$$z = (s^2 + 3)\,, s \neq 1.$$

Note 3

The R.H.S. Of (24) Is A Perfect Square For Values Of V Given By

$$v_n = n^2 + 2n + 4$$

From (24) ,We Get

$$u_n = (n^2 + 2n + 4)(n+1)$$

In View Of (23) , The Integer Solutions To (1) Are Given By

$$x_n = (n^2 + 2n + 4)(n+2),$$
$$y_n = (n^2 + 2n + 4)(n),$$
$$z_n = (n^2 + 2n + 4)\,, n = 1,2,3,...$$

Choice 6

The Option

$$x = u + v, y = u - v, z = u, u \neq v \tag{26}$$

In (1) Gives

$$3\,v^2 = u^2\,(u-1) \tag{27}$$

The R.H.S. Of (27) Is A Perfect Square When

$$u = 3s^2 + 1 \qquad (28)$$

Using (28) In 27 ,We Have

$$v = s\,(3s^2 + 1)$$

In View Of (26) , The Integer Solutions To (1) Are Given By

$$x = (3s^2 + 1)(s + 1)\ ,$$

$$y = (3s^2 + 1)(1 - s)\ ,$$

$$z = (3s^2 + 1)\ , s \neq 1.$$

Choice 7

Treating (1) As A Quadratic In X And Solving For The Same , We Have

$$x = \frac{y \pm \sqrt{4z^3 - 3y^2}}{2} \qquad (29)$$

Let

$$\alpha^2 = 4z^3 - 3y^2 \qquad (30)$$

Assume

$$z = a^2 + 3b^2 \qquad (31)$$

Write The Integer 4 In (30) As

$$4 = (1 + i\sqrt{3})\,(1 - i\sqrt{3}) \qquad (32)$$

Using (31) & (32) In (30) And Employing Factorization , Consider

$$\alpha + i\sqrt{3}\,y = (1 + i\sqrt{3})\,(a + i\sqrt{3}\,b)^3$$

On Comparing The Coefficients Of Corresponding Terms ,We Have The Values Of α, y . From (29) , The Corresponding Values To x Are Obtained.

For The Benefit Of Readers , The Two Sets Of Integer Solutions To (1) Thus Obtained Are Given Below:

$$x = a^3 - 9ab^2 - 3a^2b + 3b^3\ ,$$

Set 1: $y = a^3 - 9ab^2 + 3a^2b - 3b^3\ ,$

$$z = a^2 + 3b^2\ .$$

$$x = 6\,a^2b - 6b^3\ ,$$

Set 2: $y = a^3 - 9ab^2 + 3a^2b - 3b^3\ ,$

$$z = a^2 + 3b^2\ .$$

2. CONCLUSION

In This Paper , Varieties Of Integer Solutions To Non-Homogeneous Ternary Cubic Equation Given In Title Are Obtained. As Cubic Equations Are Plenty , One May Search For Integer Solutions To Other Forms Of Cubic Diophantine Equations.

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