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OBSERVATIONS ON "INTEGRAL SOLUTIONS OF THE NON-HOMOGENEOUS TERNARY QUINTIC EQUATION

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ABSTRACT

New choices of non-zero integer solutions to thenon-homogeneous ternary quintic equation $ax^2+by^2=(a+b)z^5, a,b>0$. Some fascinating relations between the solutions are presented.

Keywords: Non-homogeneous quintic, Ternary quintic, Integer solutions

1. INTRODUCTION

It is quite obvious that Diophantine equations are plenty. In contrast to quadratic , cubic and bi-quadratic Diophantine equations ,the quintic equations cannot be solved easily. In this context ,refer [1-4].While collecting problems on fifth degree Diophantine equations with three unknowns ,the article presented in [5] came to our reference. The authors of [5] presented only two patterns of solutions in integers. Recently , M.A.Gopalan and N.Thiruniraiselvi published a book [6] entitled "A GLIMPSE ON SPECIAL TERNARY QUINTIC DIOPHANTINE EQUATIONS WITH INTEGER SOLUTIONS" in which the quintic equation with three unknowns presented in [5] has been included as a chapter with some more sets of integer solutions .Albeit tacitly , there are other choices of fascinating integer solutions to the quintic equation with three unknowns considered in [5]. The main thrust of this paper is to obtain the new choices of integer solutions. A few relations between the solutions are presented.

Methodology The non-homogeneous ternary quintic equation under consideration is

$$ax^2+by^2=(a+b)z^5 \tag{1}$$

The process of obtaining choices of integer solutions to (1) are as below:

Choice 1

The option

$$x=ky \tag{2}$$

in (1) gives

$$(ak^2+b)y^2=(a+b)z^5$$

which is satisfied by

$$\left.\begin{aligned} y&=(ak^2+b)^2(a+b)^3\\ z&=(ak^2+b)(a+b)\end{aligned}\right\} \tag{3}$$

From (2) ,one has

$$x=k(ak^2+b)^2(a+b)^3 \tag{4}$$

Thus,(3) and (4) satisfy (1).

Choice 2

The option

$$y=kx \tag{5}$$

in (1) gives

$$(a+bk^2)x^2=(a+b)z^5$$

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which is satisfied by

$$x = (a + bk^2)^2 (a + b)^3$$
$$z = (a + bk^2)(a + b) \qquad (6)$$

From (5) ,one has

$$y = k(a + bk^2)^2 (a + b)^3 \qquad (7)$$

Thus,(6) and (7) satisfy (1).

Choice 3

The substitution

$$x = z^2 - bT, y = z^2 + aT \qquad (8)$$

in (1) leads to

$$abT^2 = z^4 (z - 1)$$

which is satisfied by

$$z = 1 + abs^2,$$
$$T = s(1 + abs^2)^2. \qquad (9)$$

From (8) ,one has

$$x = (1 + abs^2)^2 (1 - bs),$$
$$y = (1 + abs^2)^2 (1 + as). \qquad (10)$$

Thus, the values of x ,y, z given by (9) and (10) satisfy (1).

Observations

- $ax + by = (a + b)z^2$
- $asx - y + z^3 = 0$
- $bsy + x - z^3 = 0$

Note 1

In addition to (8) , one may also consider the substitution

$$x = z^2 + bT, y = z^2 - aT$$

For this option , the corresponding integer solutions to (1) are given by

$$x = (1 + abs^2)^2 (1 + bs),$$
$$y = (1 + abs^2)^2 (1 - as),$$
$$z = (1 + abs^2).$$

Choice 4

The substitution of the transformations

$$x = X - bz^2, y = X + az^2 \qquad (11)$$

in (1) leads to the non-homogeneous quintic equation

$$X^2 = z^4 (z - ab) \qquad (12)$$

After performing some algebra ,it is seen that the values of z ,X satisfying (12) are given by

$$z = z_n = ab + (s + n)^2,$$
$$X = X_n = (ab + (s + n)^2)^2 (s + n) \qquad (13)$$

From (11) ,it is obtained that

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$$x_n = (ab + (s+n)^2)^2 (s+n-b),$$
$$y_n = (ab + (s+n)^2)^2 (s+n+a). \tag{14}$$

Thus, the values of x ,y, z given by (13) and (14) satisfy (1).

Observations

- $y_n - x_n = (a+b)\, z_n{}^2$
- $(b y_n + a x_n)^2 = (z_n - ab)\,(y_n - x_n)^2$
- $(x_n + b\, z_n{}^2)^2 = (y_n - a z_n{}^2)^2 = z_n{}^4 (z_n - ab)$
- $(y_n - x_n)^2 [(x_n + b z_n{}^2)^2] = (y_n - x_n)^2 [(y_n - a z_n{}^2)^2] = (b y_n + a x_n)^2 z_n{}^4$

Note 2

In addition to (11) , one may also consider the substitution

$$x = X + bz^2 , y = X - az^2$$

For this option , the corresponding integer solutions to (1) are given by

$$x_n = (ab + (s+n)^2)^2 (s+n+b),$$
$$y_n = (ab + (s+n)^2)^2 (s+n-a) ,$$
$$z_n = (ab + (s+n)^2)$$

2. CONCLUSION

In this paper , we have presented integer solutions to the quintic equation in title which are different from the solutions given in [5,6]. As the quintic equations are plenty, one may search for other forms to quintic equations to determine their solutions in integers utilizing substitution technique and factorization method.

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