DETERMINATION OF WEIBULL PARAMETERS FOR ANNUAL AND MONTHLY WIND SPEED DISTRIBUTION

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ABSTRACT

The determination of the monthly Weibull parameters for three cities in Nigeria has been carried out in this study. The cities include Abuja, Ibadan and Jos. For each city, daily wind speed data for a period of ten years (2012 – 2021) measured at a height of 10 m was sourced from National Aeronautics and Space Administration metrological website. Analysis of the wind speed data was carried out with the use of Weibullness, a contributed package in R statistical programming software. The values of the monthly mean wind speeds estimated for each city ranges from 1.79 – 3.02 m/s, 2.35 – 4.5 m/s and 1.67 – 3.05 m/s for Abuja, Jos and Ibadan, respectively. Abuja and Jos located in the Northern part of Nigeria are considered to have higher values of monthly wind speed than Ibadan that is located in the Southern part of Nigeria. Hence, the possibility of the harvest of more wind energy in those Northern cities. In the validation of the Weibullness package, monthly percentage error obtained are seen to be insignificant as its values ranges from - 0.19 – 0.08 %, - 0.19 – 0.3 % and - 0.27 to 0.11 % for Abuja, Ibadan and Jos, respectively. Therefore, the Weibullness package is adjudged to be highly accurate and proficient in the determination of Weibull parameters.

Keywords: Weibullness, Weibull distribution, Wind energy, Wind Speed, R

1. INTRODUCTION

Energy availability is a fundamental means that drives sustainable development of a nation. It is necessary that as the population of a society increases, the energy demand have to be met both for domestic and industrial usage. To meet the ever-increasing energy demand, the use of fossil fuels continues to dominate the global energy market, thereby, increasing the annual total greenhouse gas emission [1]. More also, most economies that largely depend on the use of fossil fuels are experiencing energy poverty and loss of industrial competitiveness due to the recent high price of fossil fuel caused by the war in Ukraine and Palestine and the persistence activities of pirates on the Arabian Sea, Gulf of Aden and Red Sea. However, for most locations, the availability and accessibility of energy can be achieved by the use of alternative energy sources such as solar and wind energy that is readily available, inexpensive and environmentally friendly [2] and as the economic stance for renewable energy is absolute good as a result of the current surge in the price of fossil fuels [3]

Considering the constant relatively availability of wind during the day, wind energy is among the renewable energy sources that is presently gaining preeminence. Though, the implementation of wind energy is subjected to a number of practical difficulties and uncertainties, one of which is the intermittent and unsteady nature of wind [4]. Like the other meteorological parameters, wind usually exhibits a significant variability over a range of scales both spatially and temporally [5,6 and 7].

In order to determine the output of wind turbines in a given location during the design stage, wind energy availability is an important requirement for the evaluation of wind frequency distribution. Several statistical models have been used by different authors for the determination of wind speed frequency distribution [5 and 8]. Among these models, the two Weibull function has been found to be adequate for this purpose because it gives a good fit to the observed wind speed data both at the surface and in the upper air [9, 10 and 11].

The Weibull function has been used by researchers to analyze the wind speed data of different study locations [12-14]. As a result of the complexity of the mathematical procedure involve in the determination of the two parameters of Weibull function, the aim of this paper is to use a computer programming package to evaluate the two parameters of Weibull distribution from the wind speed data of the study locations.

2. MATERIAL

The three study locations considered in this study are sited within Nigeria. The geographical properties of the study locations are given in Table 1.

Table 1 Study locations

Study locations	Longitude (ºN)	Latitude (ºE)	Elevation (m)
Abuja	9.08	7.40	406.97

Ibadan	7.38	3.95	188.89
Jos	9.90	8.86	980.85

For each study location, a long term climatological data of daily wind speed for a period of ten years (2012 – 2021) measured at a height of 10 m is sourced from National Aeronautics and Space Administration (NASA) metrological website [15].

3. METHODOLOGY

The daily wind speed dataset obtained from NASA is prepared into monthly data frame format. Due to different numbers of days in each month, the total number of data points obtained for each data frame differ. Hence, the months are grouped into three as to obtain a uniform data frame that is acceptable by R programming software for each of the group as shown in Table 2.

Table 2 Monthly Data Frame Formation

Group	Months	Number of data points
A	Jan, Mar, May, July, Aug, Oct, Dec	310
B	Feb	283
C	April, June, Sept, Nov	300

3.1 Mathematical Analysis

The Weibull distribution is characterized by two parameter function, namely, shape parameter (k) and scale parameter (c) and it is used for the distribution of wind speed frequency. The probability density function is expressed mathematically as

$$f(V) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \qquad (1)$$

Where f(V) is the probability of observing wind speed (V), k is a dimensionless Weibull shape parameter (or factor), c is the Weibull scale parameter which has same unit with wind speed.

The cumulative distribution function which represents the probability that the wind velocity is equal to or lower than observing wind speed, or within a given wind speed range is given as

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \qquad (2)$$

To estimate Weibull parameters, several methods are available in literature [16 – 17]. These methods include; the method of moments, the method of energy pattern factor, the maximum likelihood method, the Weibull probability paper method, simple percentile estimator method and the least squares for two parameter Weibull distribution method. Due to the complexity of the integration required in dealing with the determination of the Weibull parameters [18], Weibullness, a contributed package in R statistical programming software is used for this purpose [19, 20, 21 and 22].

The monthly wind speed is an important parameter for estimation of the wind spectra availability of a location. The value of the monthly mean wind speed data is obtained from

$$v_{md} = \frac{1}{N}\left[\sum_{i=1}^{N} v_i\right] \qquad (3)$$

Where v_{md} is the monthly mean wind speed, v_i is the daily wind speed and N is number of daily wind speed.

However, in terms of Weibull density function parameters (c and k), the monthly mean wind speed value is related as

$$v_p = c\Gamma(1 + 1/k) \qquad (4)$$

Where v_p is the monthly mean wind speed calculated from the monthly values of c and k parameters of the location and Γ is gamma function

For any study location, the monthly or annual mean wind power density can be written as [23, 24 and 25].

$$\frac{P_a}{A} = \frac{1}{2}\rho c^3 \Gamma\left(1 + \frac{3}{k}\right) \qquad (5)$$

For a given period (a month or a year), the mean wind density over a given period of time (T) is the product of the mean wind power density and the period of time. It is given as [24]

$$\frac{E}{A} = \frac{1}{2}\rho c^3 \Gamma\left(1 + \frac{1}{3}\right) T \qquad (6)$$

In this study T is 8640 hours for a year.

Other important wind speed parameters in addition to the monthly mean wind speed (v_m) of Equation (3) for characterization of wind energy are the most probable wind speed which represent the most frequent wind speed for a given wind probability distribution. It is given as [23, 24 and 25].

$$v_{mp} = c\left(1 - \frac{1}{k}\right)^{1/k} \qquad (7)$$

And the wind speed of maximum energy $(v_{Max,E})$ which represents the wind speed that carries the maximum amount of wind energy. It is given as [23, 24 and 25]

$$v_{Max,E} = c\left(1 + \frac{k}{2}\right)^{1/k} \qquad (8)$$

4. RESULTS AND DISCUSSION

4.1 Monthly Mean Wind Speeds

The study locations are situated in Nigeria where two seasonal periods are prevalent; the dry and rain seasons. The dry season which usually start from November and end in April and rainy (wet) seasons that start from May to October. The dry season can be classified into three distinct periods. These are (i) The Harmattan period (December to January) when cold dry and dusty north-easterly trade winds from the Sahara Desert keep the atmosphere heavily overcast with dust for many days with characteristic hazy clear weather conditions. (ii) The dust free period (November, February, and March) which is usually characterized with high irradiation intensity and clear weather condition (iii) April which form transition period between the dust free period of February and March and the rainy season. During the rainy season each part of the country experience different level of rainfall. Though, August is characterized as month of highest rainfall. The variation in rainfall intensity thus depends on locality.

Figure 1 presents the monthly mean wind speed for the study locations. It is observed that the values of the monthly mean wind speed for the study locations vary from one month to another month. The monthly mean wind speed in Abuja and Jos ranges from 1.79 – 3.02 m/s and 2.35 – 4.50 m/s, respectively. In Figure 2, the monthly minimum and maximum wind speed in Abuja occurred in October and January, respectively. While, in Jos, the minimum and maximum wind speeds are spotted in October and January, respectively, in Figure 4. It is observed that the wind speed intensity varies from one month to the other in both study locations. The monthly wind speed curves of Abuja and Jos are seen to follow similar curve pattern with a clear depression during the months of rain season. However, with that of Jos leading while that of Abuja trail behind. This reveal that Abuja and Jos are likely to have same pattern of wind speed. However, with more wind speed intensity in Jos than in Abuja. This is expected as Abuja and Jos are situated in almost the same geographical location but positioned differently both in elevation, latitude and longitude as seen in Table 1. Remarkably, it is observed that in Abuja during the months of the rain season, there is an increase in the monthly mean wind speed in August unlike in Jos. Similar trend is also observed in Ibadan where the monthly mean wind speed ranges from 1.67 m/s in November and 3.10 m/s as shown in Figure 3. While, the curve of Ibadan is seen to follow a different pattern from those of Abuja and Jos. This may be attributed to the fact that Ibadan is located in the Southern part of Nigeria that is prone to heavy rainfall influenced by its proximity to the Atlantic Ocean compare to Abuja and Jos that are located in the Northern part of Nigeria that is relatively equidistance to the Atlantic Ocean and are usually characterized by high solar radiation intensity.
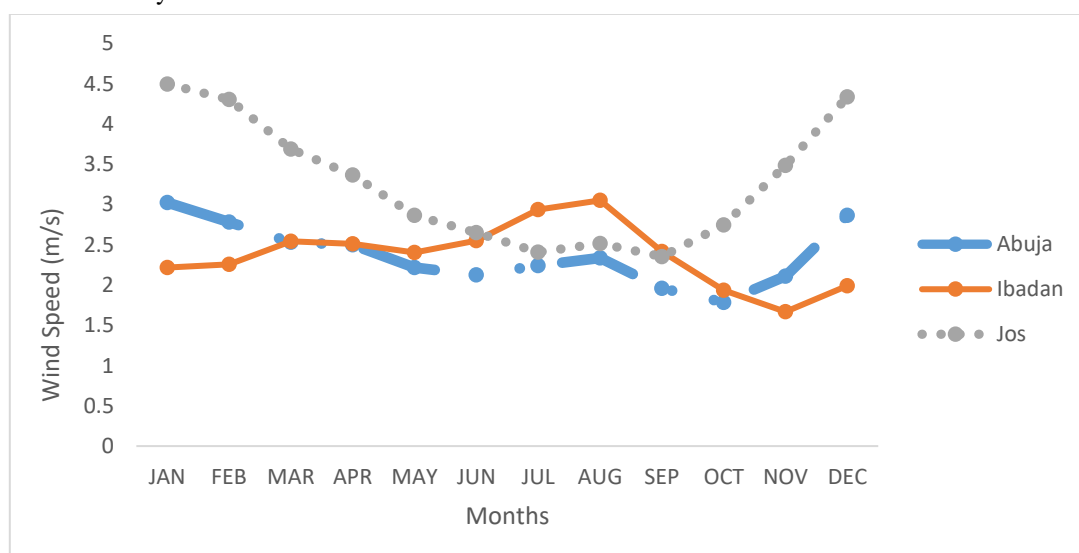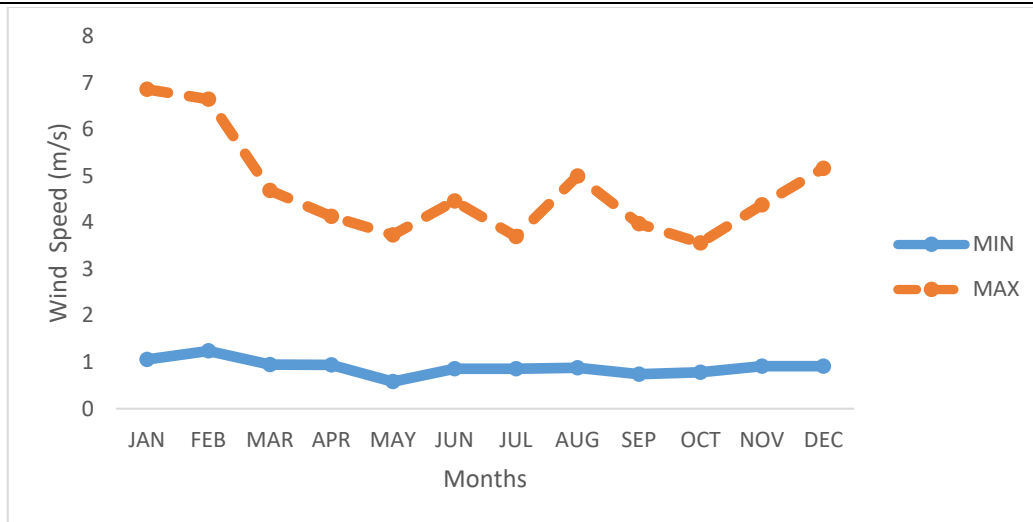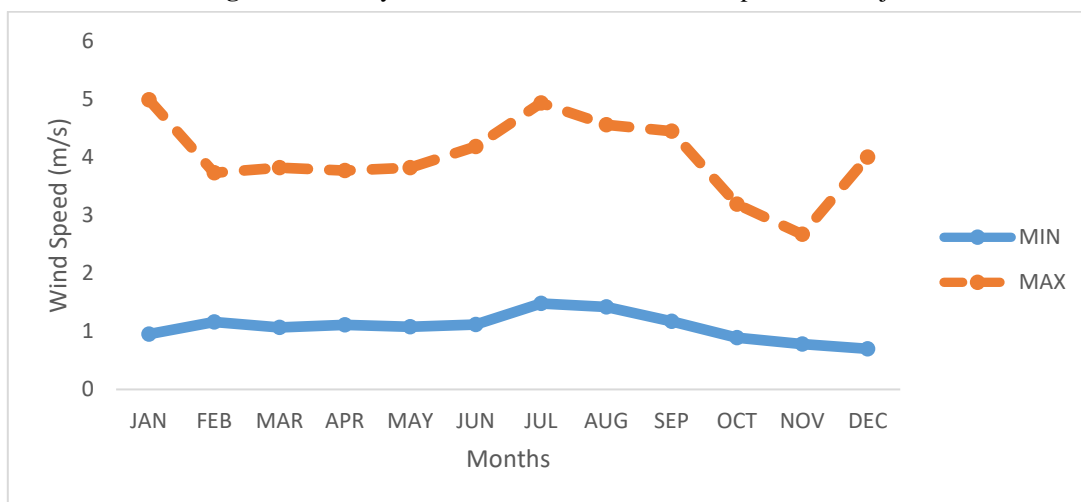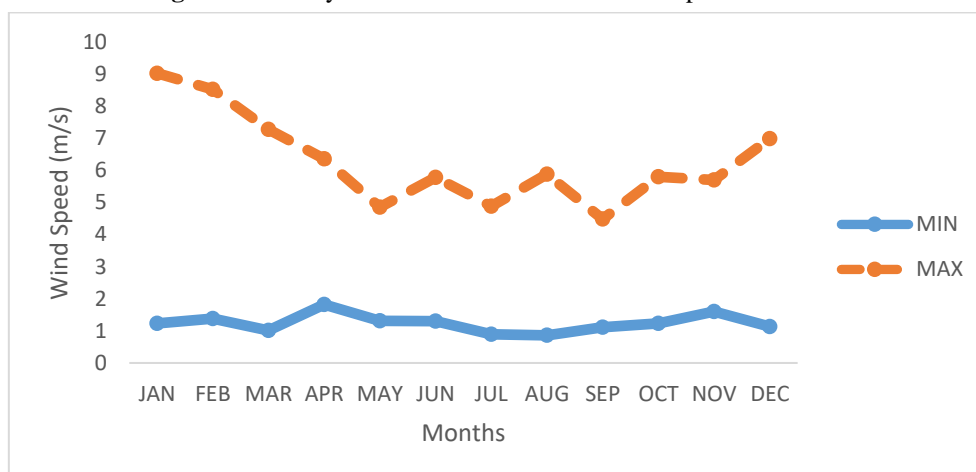

Figure 1 Monthly mean wind speed for the study locations

Figure 2 monthly minimum and maximum wind speed for Abuja

Figure 3 monthly minimum and maximum wind speed for Ibadan

Figure 4 monthly minimum and maximum wind speed for Jos

4.2 Weibull Shape and Scale Parameters

Table 3 presents the monthly and annual values of Weibull shape (k) and scale (c) parameters for the study locations

Table 3 monthly and mean yearly values of shape and scale parameters

	Abuja		Ibadan		Jos	
	k	c	k	c	k	c
Jan	4.08	3.33	4.03	2.44	3.41	5.01

Feb	4.37	3.05	5.31	2.45	3.50	4.78
Mar	5.07	2.75	6.05	2.74	3.83	4.07
April	5.03	2.72	6.72	2.70	5.10	3.66
May	4.88	2.42	6.09	2.59	5.44	3.10
June	4.85	2.32	5.62	2.76	4.87	2.89
July	4.52	2.45	6.21	3.16	4.04	2.65
Aug	4.01	2.57	6.02	3.29	3.44	2.79
Sept	3.90	2.16	4.86	2.63	4.45	2.58
Oct	4.56	1.95	5.56	2.09	4.89	2.99
Nov	3.68	2.34	5.54	1.80	5.08	3.79
Dec	4.18	3.15	3.70	2.20	4.38	4.76
Annual	4.43	2.60	5.48	2.57	4.37	3.59

The monthly values of shape and scale parameters are obtained from the daily wind speed data that spanned for ten years for each month with the use of a contributed package in R statistical programming software called Weibullness. For each study location and parameter, the mean annual values are estimated as shown in Table 4.1. The monthly values of the shape and scale parameters are observed to vary from one month to another.
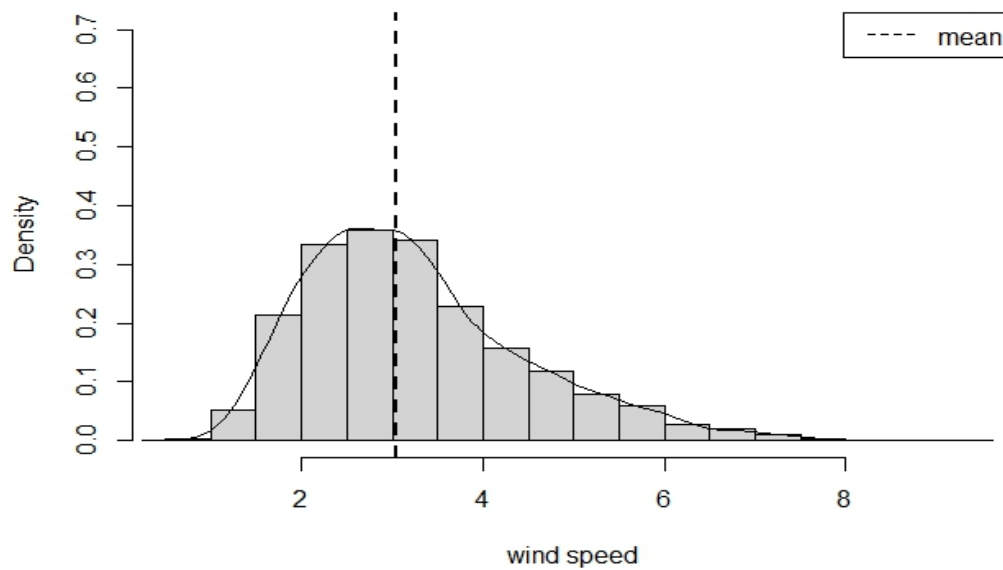
The annual wind speed density distribution is presented in Figures 5, 6 and 7 for Abuja, Ibadan and Jos, respectively. Annual mean wind speed for Abuja, Ibadan and Jos are 2.37, 2.37 and 3.26 m/s, respectively, as shown in the figures.
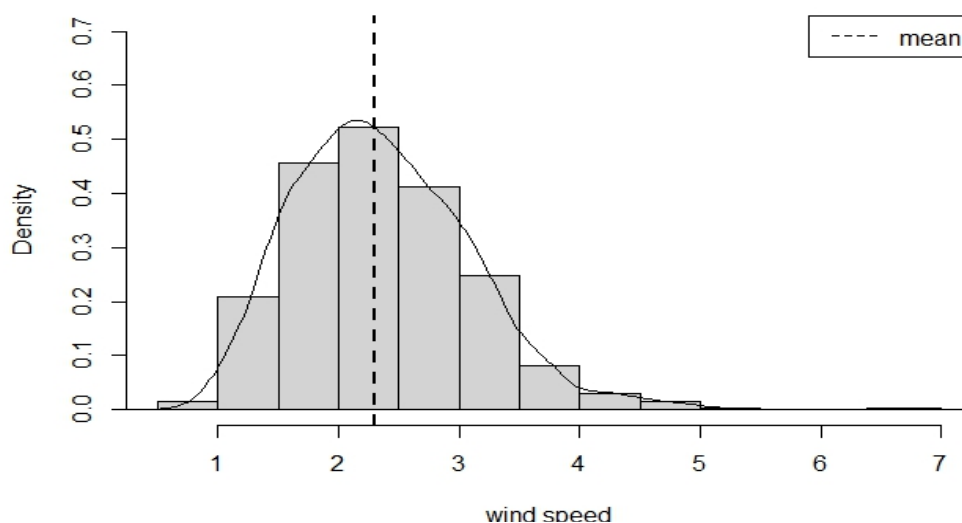
**Figure 5** annual wind speed density distribution for Abuja
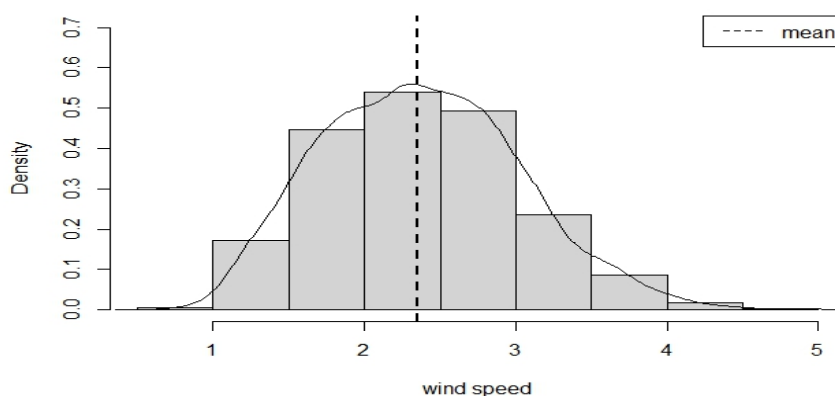
Figure 6 annual wind speed density distribution for Ibadan

Figure 7 annual wind speed density distribution for Jos

Figure 8 presents a scatter plot of daily wind speeds for the study locations. It is observed from Figure 8 that wind speed values clustered more within the range of 2 – 4 m/s for all the study location. Hence, relatively few number of days compare to days within the cluster zones are observed to have recorded outlier values of wind speeds. This is more pronounce in Jos, where higher wind speed values of outliers are recorded.
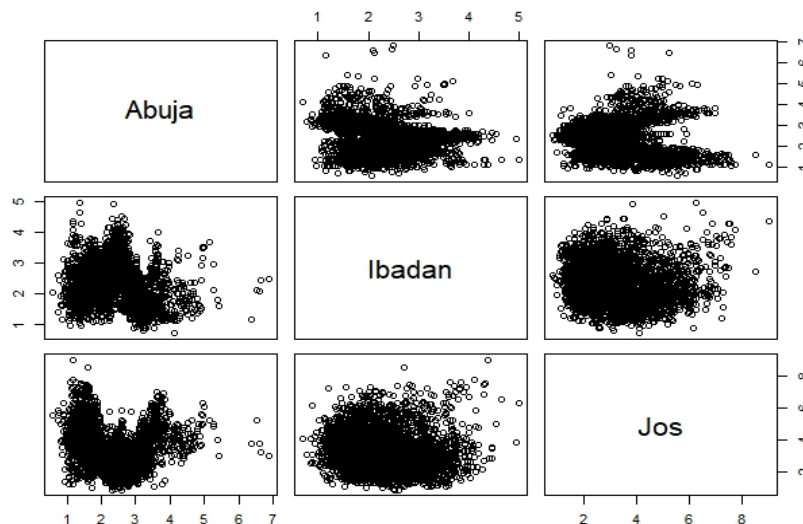
Figure 8 scatter plot of daily wind speeds for the study locations

4.3 Wind Energy Characteristics

The monthly and annual values of wind characteristic parameters for the study of wind speed availability for the study locations are presented in Table 4, 5 and 6 for Abuja, Ibadan and Jos, respectively.

Table 4 wind characteristic parameters for Abuja

	$v_{md}(\frac{m}{s})$	$v_p(\frac{m}{s})$	$v_{mp}(\frac{m}{s})$	$v_{max,E}(\frac{m}{s})$	P/A $(\frac{W}{m^2})$	E/A $(\frac{Wh}{m^2})$
JAN	3.02	3.02	3.11	3.48	20.68	178667.04
FEB	2.78	2.78	2.87	3.28	15.73	135805.94
MAR	2.53	2.53	2.64	3.09	11.39	98446.99
APR	2.50	2.50	2.61	3.06	11.06	95534.96

	$v_{md}(\frac{m}{s})$	$v_p(\frac{m}{s})$	$v_{mp}(\frac{m}{s})$	$v_{max,E}(\frac{m}{s})$	P/A $(\frac{W}{m^2})$	E/A $(\frac{Wh}{m^2})$
MAY	2.22	2.22	2.31	2.85	7.75	66981.37
JUN	2.13	2.12	2.21	2.78	6.82	58890.97
JUL	2.24	2.24	2.32	2.86	8.14	70304.59
AUG	2.34	2.33	2.39	2.92	9.57	82721.28
SEP	1.96	1.95	2.00	2.65	5.68	49108.60
OCT	1.79	1.78	1.85	2.56	4.11	35479.59
NOV	2.11	2.11	2.14	2.75	7.29	63019.16
DEC	2.87	2.86	2.95	3.35	17.48	151035.82
Annual	2.37	2.37	2.45	2.97	10.48	90499.69

Table 5 wind characteristic parameters for Ibadan

	$v_{md}(\frac{m}{s})$	$v_p(\frac{m}{s})$	$v_{mp}(\frac{m}{s})$	$v_{max,E}(\frac{m}{s})$	P/A $(\frac{W}{m^2})$	E/A $(\frac{Wh}{m^2})$
JAN	2.22	2.21	2.27	2.83	8.15	70380.38
FEB	2.25	2.25	2.35	2.88	7.97	68842.12
MAR	2.54	2.54	2.66	3.11	11.13	96146.13
APR	2.51	2.52	2.63	3.08	10.64	91949.77
MAY	2.40	2.40	2.51	3.00	9.40	81180.01
JUN	2.55	2.55	2.66	3.11	11.41	98565.35
JUL	2.94	2.93	3.07	3.43	17.04	147214.93
AUG	3.05	3.05	3.19	3.52	19.29	166701.01
SEP	2.41	2.41	2.51	2.99	9.99	86310.32
OCT	1.93	1.93	2.02	2.67	4.98	43031.65
NOV	1.67	1.67	1.74	2.52	3.19	27553.16
DEC	1.99	1.99	2.02	2.67	6.11	52797.45
Annual	2.37	2.37	2.47	2.98	9.94	85889.36

Table 6 wind characteristic parameters for Jos

	$v_{md}(\frac{m}{s})$	$v_p(\frac{m}{s})$	$v_{mp}(\frac{m}{s})$	$v_{max,E}(\frac{m}{s})$	P/A $(\frac{W}{m^2})$	E/A $(\frac{Wh}{m^2})$
JAN	4.49	4.50	4.52	4.81	73.36	633793.83
FEB	4.30	4.30	4.34	4.63	63.27	546636.33
MAR	3.69	3.68	3.76	4.07	38.23	330305.99
APR	3.36	3.36	3.50	3.79	26.71	230742.76
MAY	2.86	2.86	2.99	3.36	16.24	140282.82
JUN	2.65	2.65	2.75	3.18	13.18	113913.11
JUL	2.41	2.41	2.47	2.97	10.48	90559.74
AUG	2.52	2.51	2.53	3.05	12.67	109434.43
SEP	2.35	2.36	2.44	2.94	9.54	82444.08
OCT	2.77	2.74	2.85	3.26	14.67	126744.41
NOV	3.48	3.48	3.63	3.90	29.71	256722.02
DEC	4.33	4.34	4.49	4.67	59.80	516645.57
Annual	3.27	3.26	3.36	3.72	30.65	264852.09

4.4 Validation of Weibull Parameters

To validate the R programming contributed package called Weibullness used in this study to determine the values of k and c as given in Table 3. Percentage error is estimated from the following parameters as

$$\% \text{ error} = \frac{v_p - v_{md}}{v_{md}} \text{ x } 100 \qquad (4.1)$$

From Figure 9, it is observed that in each study location a positive or negative values of percentage error is determined for each month. In Abuja, the percentage error varies from -0.19 to 0.08 % annual. While, the annual range of percentage error for Ibadan and Jos varies from -0.19 to 0.3 % and -0.27 to 0.11 %, respectively. The variation may be partially attributed to approximation of number i.e. round up and down of numbers from one software to another. Hence, considering the range of the percentage error estimated between the monthly mean wind speed of the wind speed data and that from the Weibull parameters. It is adjudged that the Weibullness R contributed package performed excellently well in the determination of the Weibull parameters.
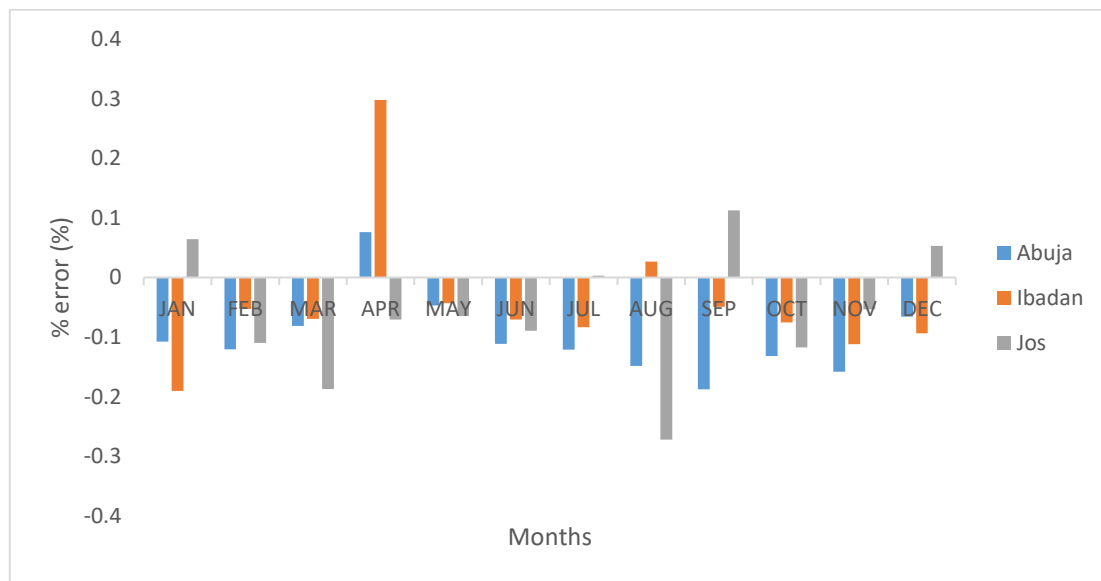

Figure 9 Percentage error of monthly mean wind speed

5. CONCLUSION

The monthly and annual c and k parameters of the Weibull function for Abuja, Ibadan and Jos have been estimated in this study with the use of Weibullness, a contributed package in R statistical package. The percentage error validation carried out on Weibullness using the wind speed data of the study locations shows that the package performed excellently well. The variation in the percentage error may be attributed to data migration from one software to another. Outliers values of wind speed are seen to be heavily present in wind speed data and such values require special attention in the design analysis of wind conversion systems.

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