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GB*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we have introduced and analyze a new class of Gβ*-continuous map in grill topological space. Its relation to various other continuous functions are investigated.

Keywords: Gβ*- closed sets, Gβ*- continuous maps,

2020 Mathematics Subject Classification: 54C05, 54C08.

1. INTRODUCTION

Choquet introduced the concept of grills on a topological space [3]. Hatir and Jafari [6] obtained a new decomposition of continuity in terms of grills. Antony Rex Rodgio and Jessie Theodore [2] introduced β*- continuous maps in topological space using β*- closed sets. In this paper, we have introduced and analyze a new class of Gβ*-continuous map in grill topological space (X, τ, G).

2. PRELIMINARIES

2.1 Definition: [3] A non-null collection G of subsets of a topological spaces X is said to be a grill on X if

- $\phi \notin G$
- A ∈ G and A ⊆ B ⟹B ∈ G,
- A, B ⊆ X and A ∪ B ∈ G ⟹ A ∈ G or B ∈ G.

2.2 Definition: Let (X, τ, G) be a grill topological space. A subset A in X is said to be

- ϕ -open [6] if A ⊆ Int(ϕ(A)),
- Gα-open[1] if A ⊆ Int(Ψ(Int(A))),
- G-preopen [6] if A ⊆ Int(Ψ(A)),
- G-semi-open [1] if A ⊆ Ψ(Int(A)),
- Gβ –open [1] if A ⊆ Cl(Int(Ψ(A))).

The family of all Gα-open (resp. G-preopen, G-semi-open, Gβ -open) sets in a grill topological space (X, τ, G) is denoted by GαO(X) (resp. GPO(X), GSO(X), GβO(X)).

2.3 Definition:[12] A subset A of X is called a Gβ*-closed set if Gβcl(A) ⊆ int(U) whenever A⊆ U and U is ω-open in X.The class of all Gβ*-closed sets in X is denoted by Gβ*C(τ). That is Gβ*C(τ)={A⊂ X: A is Gβ*-closed in X}.

2.4 Definition: A function f : $(X, \tau, G_X) \to (Y, \sigma, G_Y)$ is said to be

- grill α-continuous[1] if the inverse image of each open set of Y is Gα-open.
- grill semi –continuous[1] if the inverse image of each open set of Y is G-semi-open
- grill pre - continuous [6] if the inverse image of each open set of Y is G-preopen.
- grill β –continuous [1] if the inverse image of each closed set of Y is Gβ –closed.
- grill g - continuous [7] if the inverse image of each closed set of Y is Gg-closed.
- grill r g - continuous [1 0] if the inverse image of each closed set of Y is Grg-closed.
- grill gsp –continuous[12] if the inverse image of each closed set of Y is Ggsp-closed.
- grill ω – continuous [12] if the inverse image of each closed set of Y is Gω-closed.
- grill *g – continuous[12] if the inverse image of each closed set of Y is G *g-closed
- grill g* - continuous[11] if the inverse image of each closed set of Y is Gg*-closed.
- grill sg –continuous[9] if the inverse image of each closed set of Y is Gsg-closed.
- grill gs-continuous [1 3] if the inverse image of each closed set of Y is Ggs-closed

- grill pre-semi-continuous[12] if the inverse image of each closed set of Y is G-pre-semi closed
- grill $\hat{\eta}^*$-continuous[12] if the inverse image of each closed set of Y is G$\hat{\eta}^*$-closed

3. GB*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES

3.1 Definition: A map f: X→Y is called Gβ*- continuous if f^{-1}(V) is Gβ*- closed in X for every closed set V in Y.

3.2 Proposition: Every continuous (respectively Gα-continuous, G-semi continuous) map is Gβ*- continuous but not conversely.

Proof: The proof follows from the fact that every closed (respectively Gα- closed, G- semi closed) set is Gβ*-closed.

The converses of Proposition is shown by the following example

3.3 Example: Let X = {a, b, c}, Y = {p, q}, τ = {ϕ, {a}, X}, σ = { ϕ,{q},Y}, G_X = {{a},{a, b}, {a, c}, X} and G_Y = {{q}, Y}. The map f: X→Y define by f(a) = f(b) = p and f(c)=q is Gβ* continuous. However f is none of G-semi-continuous, Gα-continuous and continuous since for the closed set U= {p} in Y, f^{-1}(U) = {a, b} is none of G-semi closed, Gα-closed and closed in (X, τ, G_X) but it is Gβ* closed in (X, τ, G_X).

3.4 Proposition: Every Gβ*-continuous map is Ggsp-continuous (resp. G$\hat{\eta}^*$- continuous) but not conversely.

Proof: Since every Gβ*-closed set is Ggsp-closed (resp.G$\hat{\eta}^*$-closed)

3.5 Example: Let X={a, b, c, d}, Y={p, q, r}, τ={ ϕ,{a, b}, X}, σ= { ϕ,{p},{r},{p, r},Y}, G_X = {{a},{a, b}, {a, c},{a, d}, {a, b, c},{a, b, d},{a, c, d}, X} and G_Y = {{q},{p,q},{q, r}, Y}. Define f: (X, τ, G_X) →(Y, σ, G_Y) by f(a)=f(d)=p, f(b)=q, f(c)=r. Then it can be seen that f is Ggsp-continuous and G$\hat{\eta}^*$-continuous, but not Gβ*continuous, because for the closed set {q} in Y, f^{-1}({q}) = {b} is not Gβ*- closed in (X, τ, G_X).

3.6 Proposition: The following examples show that Gβ*-continuity is independent of Gω-continuity and Grg-continuity.

3.7 Example: Let X=Y= {a, b, c}, τ= { ϕ, {a},{b},{a, b}, X}, σ={ ϕ,{a, c},Y}, G_X = {{b}, {a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Here Gβ*c(τ)=P(X)-{a, b}. Then f is Gβ*-continuous but none of Gω- continuity and Grg-continuous, since {b} is closed in Y but f^{-1}({b}) = {b} is none of Gω- closed and Grg-closed in (X, τ, G_X).

3.8 Example: Let X = Y = {a, b, c}, τ = { ϕ,{a},{b, c},X}, σ = { ϕ,{a, c},Y},

G_X = {{b},{a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Here Gβ*c(τ)= {ϕ, {a},{b, c}, X}. Then f is Gω-continuous and Grg continuous but not Gβ*-continuous for f^{-1}({b})={b} is not Gβ*-closed in (X, τ, G_X).

3.9 Proposition: The following examples, show that Gβ*-continuity is independent of Gg-continuity, Gsg-continuity and Ggs-continuity.

3.10 Example: Let X= {a, b, c}=Y , τ={ ϕ,{a},{b, c},X}, σ={ ϕ,{b}, Y}, G_X = {{b},{a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is Gg-continuous, Ggs-continuous and Gsg-continuous, but not Gβ*-continuous because for the closed set {a, c}, f^{-1} ({a, c}) = {a, c} is not Gβ*-closed in (X, τ, G_X).

3.11 Example: Let X=Y= {a, b, c}, τ= { ϕ, {a}, {b}, {a, b}, X}, σ={ ϕ,{a, c},Y}, G_X = {{b},{a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is Gβ*-continuous but none of Gg- continuous, Gsg-continuous and Ggs-continuous because for the closed set {b} in Y, f^{-1}({b}= {b}is Gβ*-closed but none of Gg-closed, Gsg-closed and Ggs-closed.

3.12 Proposition: The following examples show that Gβ*-continuity is independent of G-pre-continuity, G-semi-pre-continuity and G-pre-semi-continuity.

3.13 Example: Let X=Y= {a, b, c, d}, τ= { ϕ, {a, b}, X}, σ= { ϕ, {c}, Y}, G_X = {{b},{a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X} and G_Y = {{a},{a, b},{a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is Gβ*-continuous but none of G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, since {a, b, d} is closed in Y but f^{-1}({a, b, d}) = {a, b, d} is none of G-preclosed, G-semi-preclosed and G-pre-semiclosed in (X, τ, G_X).

3.14 Example: Let X= {a, b, c}=Y , τ={ ϕ,{a},{b, c},X}, σ={ ϕ,{c}, Y}, G_X = {{b},{a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, but not Gβ*-continuous because for the closed set {a, b}, f^{-1} ({a, b}) = {a, b} is not Gβ*-closed in (X, τ, G_X).

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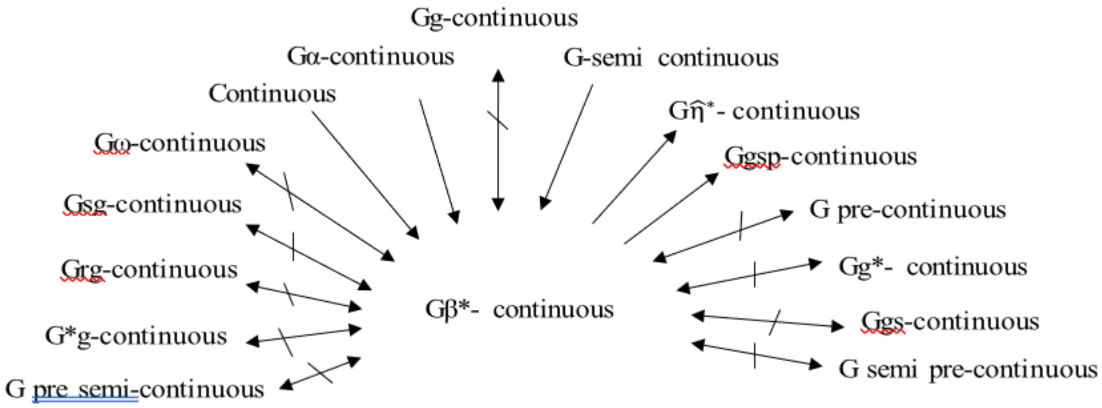
3.15 Proposition: The following examples show that Gβ*-continuity is independent of G*g-continuity and Gg*-continuity.

3.16 Example: Let X= {a, b, c}=Y, τ = { ϕ, {a}, {b}, {a, b}, X}, σ= { ϕ, {a, c}, Y}, G_X = {{b}, {a, b}, {b, c}, X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is Gβ*-continuous but neither Gg*- continuous nor G*g-continuous because for the closed set {b} is Gβ*-closed but neither Gg*-closed nor G*g-closed in (X, τ, G_X).

3.17 Example: Let X= {a, b, c, d}, Y= {a, b, c}, τ = { ϕ, {a, b}, X}, σ= { ϕ, {a, c}, Y}, G_X={{a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d},X} and G_Y = {{a},{a, b},{a, c}, Y}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map. Then f is Gg*- continuous and hence G*g-continuous but not Gβ*-continuous because for the closed set {b} is Gg*-closed and hence G*g-closed but not Gβ*-closed in (X, τ, G_X).

From the above discussion are shown in the following implications:



3.18 Proposition: The composition of two Gβ*-continuous maps need not be Gβ*-continuous.

3.19 Example: Let X=Y={a, b, c}, Z={p, q}, τ={ ϕ,{a},{b},{a, b}, X}, σ={ ϕ, {a}, Y}, η={ ϕ,{p}, Z} G_X = {{b},{a, b}, {b, c}, X}, G_Y = {{a},{a, b}, {a, c}, Y} and G_Z = {{p}, Z}. Let f: (X, τ, G_X) →(Y, σ, G_Y) be the identity map and define a map g: (Y, σ, G_Y) →(Z, η, G_Z) by g(a)=g(b)=q and g(c)=p. Then both f and g are Gβ*- continuous. Consider the closed set A= {q} in (Z, η). For this set $(g \circ f)^{-1}$(A) = f^{-1} (g^{-1}(A)) = {a, b} is not Gβ*-closed in (X, τ, G_X). Therefore g ∘ f is not Gβ*-continuous.

3.20 Proposition: If f: X→Y is Gβ*-continuous and g: Y→Z is continuous, then their composition g ∘ f: X→Z is Gβ*-continuous.

Proof: Clearly follows from definitions.

3.21 Theorem: A map f: X→Y is Gβ*- continuous if and only if f^{-1}(U) is Gβ*-open for every open set U in Y.

Proof: Let f: X→Y be Gβ*-continuous and U be an open set in Y. Then $f^{-1}(U^C)$ is Gβ* closed in X. But $f^{-1}(U^C) = (f^{-1}(U))^C$ and so f^{-1}(U) is Gβ*-open in X. Converse is similar.

4. CONCLUSION

In this paper, we have introduced and analyzed Gβ*-continuous map in grill topological space. Its relation to various other continuous functions are investigated. Also verified the composition of two Gβ*-continuous maps need not be Gβ*-continuous.

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